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Daily average sulfur dioxide in Greater Vitória Region: a space-time analysis

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Abstract

This study explores the class of Space-Time Autoregressive Moving Average (STARMA) models in order to describe and identify the behavior of SO\textsubscript{2} daily average concentrations observed in the Greater Vitória Region (GVR), Brazil. These models are particularly useful in modeling atmospheric pollution data owing to the complex pollutant dispersion dynamics at temporal and spatial scales.

The data were obtained at the air quality monitoring network of GVR, recorded from January 2005 to December 2009. Our findings indicate that SO\textsubscript{2} daily averages tended to be higher than the guidelines suggested by the World Health Organization (daily average of 20 µg/m\textsuperscript{3}), for almost all the analyzed sites. The time series obtained for each monitoring station show high variability, mostly caused by some atypical values observed during the period. The main fluctuations in the data are caused by cyclical components, which change from one to another station. On the whole, the cycles are not only weekly (as expected, due to the daily measurements) but also monthly and seasonal.

Resampling bootstrap techniques were used in order to handle the lack of the distributional assumptions made for fitting the model. The obtained bootstrap prediction intervals showed to be much larger than the intervals obtained under the Gaussian distribution assumption.

The fitted STARMA model indicated that the permanence time of SO\textsubscript{2} in GVR atmosphere is around 3–4 days. During the period observed, the pollutants released in a site disperse over a large expanse of the region, influencing SO\textsubscript{2} concentrations observed in the vicinity. The quality of the adjusted model suggests that the model is able to predict in-sample values, as well as to forecast average concentrations for one day in advance with good reliability.

Keywords: air pollution, bootstrap, forecasting, space-time models, STARMA, sulfur dioxide.

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1. Introduction

The GVR is located on the Brazilian South Atlantic coast in the state of Espírito Santo (ES) and comprises of seven main cities, including the capital Vitória. Its population has grown significantly in the last decades as a consequence of rapid industrialization. The increase of the industrial activities, as well as the constant growth of traffic (almost 50% increases from 2001 to 2011), has caused a large impact on the atmospheric quality in the area.

Particularly, sulfur dioxide (SO$_2$) is considered to be the major indicator of the industrial activities in the area, where the mining and iron, as well as the steel industries, contribute with almost 76% of SO$_2$ released to the atmosphere (Instituto Estadual de Meio Ambiente e Recursos Hídricos [IEMA], 2011). An overall view of the air quality parameters in GVR shows that SO$_2$ levels do not exceed the standard levels established by the Brazilian law and there have not been any reported air pollution alerts due to this pollutant. However, according to the Instituto Brasileiro de Geografia e Estatística [IBGE] (2012), in 2010, Vitória was the city with the highest annual SO$_2$ average in Brazil.

Sulfur dioxide is the main precursor of acid rain and sulfuric acid smog pollution. At the same time, it can be oxidized in the atmosphere to form sulfate aerosol, which is an important component of fine particles suspended in the urban atmosphere. Its reaction with other major atmospheric pollutants such as nitrogen oxide can also affect the atmospheric concentrations of these pollutants. Therefore, SO$_2$ is a significant contributor to the quality of the environment (Yang et al., 2009).

In view of this pollution problem, it is important to develop statistical models for diagnosis and short-term prediction in order to provide accurate early warnings for the air quality control. As pointed out by McCollister and Wilson (1975), there is also the possibility that foreknowledge of high pollution potential could be used to reduce future atmospheric pollutant concentrations through timely reduction of emissions by traffic control or industrial shut-down.

Several statistical modeling approaches have been proposed to describe trends and forecasting SO$_2$ levels (Brunelli et al. (2007, 2008), Castro et al. (2003), Chelani
et al. (2002), Lalas et al. (1982), Nunnari et al. (2004), Perez (2001), Roca Pardiñas et al. (2004), Tecer (2007), among others). The most of forecasting statistical models for SO₂ is based on univariate time series approaches. For example, Cheng and Lam (2000), Hassanzadeh et al. (2009), Kumar and Goyal (2011), Lalas et al. (1982), McCollister and Wilson (1975), Schlink et al. (1997). As explained by Turalioglu and Bayraktar (2005), such models are incapable of providing regional information on the spatial variations of air pollutants.

Some other researches have modeled the spatial scale and used data reduction methods like principal component analysis to summarize the regional variation of SO₂ (Ashbaugh et al. (1984), Beelen et al. (2009), Ibarra Berástegui et al. (2009), de Kluizenaar et al. (2001), Kurt and Oktay (2010), Zou et al. (2009)). However, many of these spatial approaches do not account for the serial autocorrelation latent in data measured over time.

Considering that the data used in the majority of the air pollution studies are obtained from air quality monitoring networks, where the concentrations are observed over various spatial locations along time, it is reasonable to model time and space scales simultaneously aiming to capture explicitly the inherent uncertainty of the air pollution type data. Particularly, for SO₂ studies see Fan et al. (2010), Rouhani et al. (1992), Turalioglu and Bayraktar (2005), Yu and Chang (2006) and Zeri et al. (2011) among others.

In this context, the class of the space-time models is quite effective, allowing the practician to obtain accurate forecasts of the pollution events and to interpolate the spatial regions of interest. One of the most useful approaches of this kind of models, yet less explored in air pollution studies, is the class of STARMA models. This approach is an extension of the classic univariate ARMA time series models into the spatial domain, where the observations at each location at a fixed time are modeled as a weighted combination of past observations at different locations.

Our aim here is to explore the class of STARMA models as an alternative methodology to describe the dynamics of sulfur dioxide dispersion and to obtain short-term forecasts of SO₂ daily average in GVR, which can be used to direct new standards
for air quality management policies and emission control at specific locations.

This paper is outlined as follows: Section 2 presents the main characteristics of the region under the study as well as the description of the analyzed data. The three-stage procedure for STARMA modeling is also introduced in this section. Section 3 describes the data processing and the results obtained for the fitted STARMA model. Section 4 closes with a brief summary of the results obtained from the application of the model.

2. Data and methodology

2.1. Study area

The GVR is located in the Brazilian South Atlantic coast (latitude 20°19S, longitude 40°20W). The climate is tropical humid with average temperatures ranging from 23°C to 30°C. The rainfall occurs mainly from October to January, with annual precipitation volume higher to 1400 mm.

Its topography varies from plains to mountain range interspersed with small and medium size rocky massif, which favors the flowing of the humid winds from the sea (Instituto Jones dos Santos Neves [IJSN], 2012). Therefore, the dispersion of the pollutants is also favored over a large area of the region. Its main atmospherical flowing systems are the South Atlantic subtropical anticyclone, which causes the predominant eastern and northeastern winds, and the moving polar anticyclone, responsible for the cold fronts from the southern region of the continent, characterized by low temperatures, mist and strong winds (Instituto Estadual de Meio Ambiente e Recursos Hídricos [IEMA], 2007).

The region is constituted by seven main cities: Vitória (capital city of ES), Serra, Vila Velha, Cariacica, Viana, Guarapari and Fundão. These cities take almost half of total population of Espírito Santo State (48%) and 57% of the urban population in the State (Instituto Brasileiro de Geografia e Estatística [IBGE], 2012). According to the IJSN, the region occupies only 5% of ES territory, however its population density is nine times higher to the overall mean of State. Besides, it produces 58% of the wealth and consumes 55% of the total electric power produced in the State.
The GVR has two of the major seaports in Brazil: Vitória Port (located in downtown) and Tubarão Port (located at the North region of Vitória). The main industrial activities of GVR are related to iron and steel industry, stone quarry, cement and food industries, among others. These activities represent nearly 55% to 65% of the total potentially pollutant fonts in the State (IEMA, 2011).

In view of the increasing deterioration of the air quality, the IEMA installed the Automatic Air Quality Monitoring Network (AAQMN) of GVR in 2000. Currently, the network is composed of nine monitoring stations (the last one started operations in September 2012), all of them located in strategic urban areas (see Figure 1). The network measures continuously some meteorological variables as well as the concentration of the pollutants: particular matter, fine particles $< 10\mu m$ (PM$_{10}$), sulfur dioxide (SO$_2$), carbon monoxide (CO), nitrogen oxides (NO$_x$), ozone (O$_3$) and hydrocarbons (HC).

2.2. Data

We analyzed daily average SO$_2$ concentration ($\mu g/m^3$) data from January 1, 2005 to December 31, 2009, obtained from seven AAQMN monitoring stations. The main sources of pollutants of each monitoring station are summarized in Table 1. Aiming
to ensure the reliability of our study, the monitoring stations having more than 30% missing values for the full analyzed period were discarded. Except for Jardim Camburi station (36% missing values), all the stations met the criterion for inclusion in the study.

Table 1: Description of the AAQMN monitoring stations in GVR.

<table>
<thead>
<tr>
<th>Monitoring station</th>
<th>Main sources</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laranjeiras</td>
<td>Industrial and traffic</td>
<td>40°15’24.74”W</td>
<td>20°11’26.88”S</td>
</tr>
<tr>
<td>Jardim Camburi</td>
<td>Industrial and traffic</td>
<td>40°16’06.49”W</td>
<td>20°15’15.03”S</td>
</tr>
<tr>
<td>Enseada do Suá</td>
<td>Port of Tubarão and traffic</td>
<td>40°17’26.92”W</td>
<td>20°18’43.29”S</td>
</tr>
<tr>
<td>Vitória Centro</td>
<td>Traffic, seaports, Industrial</td>
<td>40°20’13.87”W</td>
<td>20°19’09.42”S</td>
</tr>
<tr>
<td>Ibes</td>
<td>Traffic and industrial</td>
<td>40°19’04.38”W</td>
<td>20°20’53.47”S</td>
</tr>
<tr>
<td>Vila Velha Centro</td>
<td>Traffic and industrial</td>
<td>40°17’37.77”W</td>
<td>20°20’04.81”S</td>
</tr>
<tr>
<td>Cariacica</td>
<td>Traffic and industrial</td>
<td>40°24’01.59”W</td>
<td>20°20’29.92”S</td>
</tr>
</tbody>
</table>

Font: IEMA

The missing values were filled using the Gibbs sampling for multiple imputations of the incomplete multivariate data suggested by Aerts et al. (2002). This algorithm imputes an incomplete column (in our case, each column corresponds to a monitoring station) by generating plausible synthetic values given the other columns in the data. Each incomplete column must act as a target column, and has its own specific set of predictors. The default set of predictors for a given target consists of all other columns in the data set. All these computations were made using the language and environment for statistical computing R 2.15.2 (R Core Team, 2012).

Once the database was filled, we calculated the 24-hour average concentrations. Therefore, the analyzed database contains 1826 observations for the six monitoring stations.
(sites) considered here. The first 1811 observations were used for modeling purposes and the last 15, corresponding to the last two weeks of the full period, were used for forecasting purposes.

2.3. The STARMA Model

Spatial time series can be viewed as time series collected simultaneously in a number of fixed sites with fixed distances between them. As pointed out by Subba Rao and Antunes (2003), the space-time models are used to explain the dependence along time in situations that present systematic dependence between observations in several sites.

The class of STARMA models was developed by Pfeifer and Deutsch (1980b). The processes which can be represented by STARMA models are characterized by a single random variable $Z_i(t)$, observed at $N$ fixed spatial locations ($i = 1, 2, \ldots, N$) on $T$ time periods ($t = 1, 2, \ldots, T$). The $N$ spatial locations can represent several situations, like states of a country or regions with monitoring stations inside a city, for example.

The dependence between the $N$ time series is incorporated into the model through hierarchical weighting $N \times N$ matrices, specified before the data analysis. These matrices must include the relevant physical characteristics of the system into the model, as for example, the distance between the center of several cities or the distance between monitoring stations from a monitoring network (Kamarianakis and Prastacos, 2005).

As in the case of univariate time series, observations $z_i(t)$ from the process $\{Z_i(t)\}$, are expressed in terms of a linear combination of previous observations and errors at the site $i = 1, 2, \ldots, N$. In this case, due to the spatial dependence of the system, the model must incorporate also past observations and errors from the neighboring spatial orders. In this paper, the first order neighbors are those sites which are closer to the location of interest, the second order neighbors are those more distant than the first ones, even less distant than the third order neighbors, and so on.

The STARMA model, denoted by STARMA($p_{\lambda_1, \lambda_2, \ldots, \lambda_p}; q_{m_1, m_2, \ldots, m_q}$), can be represented by the matrix equation:

$$
\mathbf{z}(t) = - \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} \mathbf{W}^{(l)} \mathbf{z}(t-k) + \sum_{k=1}^{q} \sum_{l=0}^{m_k} \theta_{kl} \mathbf{W}^{(l)} \mathbf{\varepsilon}(t-k) + \mathbf{\varepsilon}(t),
$$

where $\mathbf{z}(t) = [z_1(t), \ldots, z_N(t)]'$ is a $N \times 1$ vector of observations at time $t = 1, \ldots, T$, $p$ represents the autoregressive order (AR), $q$ represents the moving average order (MA), $\lambda_k$
is the spatial order of the $k$–th AR term, $m_k$ is the spatial order of the $k$–th MA term, $\phi_{kl}$ and $\theta_{kl}$ are the parameters at temporal lag $k$ and spatial lag $l$, $\mathbf{W}(l)$ is the $N \times N$ weighting matrix for the spatial order $l$, with diagonal entries 0 and off-diagonal entries related to the distances between the sites. By definition, $\mathbf{W}(0) = \mathbf{I}_N$ and each row of $\mathbf{W}(l)$ must add up to 1. It is assumed that $\mathbf{\varepsilon}(t) = [\varepsilon_1(t), \ldots, \varepsilon_N(t)]'$, the random error vector at time $t$, is a weakly stationary Gaussian process, with

$$E[\mathbf{\varepsilon}(t)] = \mathbf{0},$$

$$E[\mathbf{\varepsilon}(t)\mathbf{\varepsilon}'(t + s)] = \begin{cases} 
G, & \text{if } s = 0 \\
0, & \text{otherwise},
\end{cases}$$

$$E[\mathbf{z}(t)\mathbf{\varepsilon}'(t + s)] = 0, \quad \text{for } s > 0,$$

where $E(\cdot)$ is the expected value of the variable.

There are two subclasses of the model in Equation 1: STAR($p_1, \ldots, p_p$) when $q = 0$ and STMA($q_{m_1}, m_2, \ldots, m_q$) when $p = 0$. The stationarity condition is based on:

$$\det \left( \mathbf{I}_N + \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} \mathbf{W}(l) x^k \right) \neq 0,$$

for $|x| \leq 1$. This condition determines the region of $\phi_{kl}$ values for which the process is weakly stationary.

As explained by Deutsch and Pfeifer (1981), the proper approach to estimation is highly dependent upon the nature of the variance-covariance matrix of the errors. If $\mathbf{G}$ is assumed to be diagonal, the model estimation should proceed using weighted least squares method. In particular, when the processes for all the $N$ sites have the same variance ($\mathbf{G} = \sigma^2 \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix), the estimation technique reduces to ordinary least squares.

Lastly, when $\mathbf{G}$ is not diagonal, estimation should be performed using generalized least squares. The authors develop procedures for testing hypotheses about $\mathbf{G}$ and provide tables of the critical values for the proposed tests.

The covariance between the $l$ and $k$ order neighbors at the time lag $s$ is defined as space-time covariance function (STCOV). Let $E[Z(t)] = 0$, the STCOV can be expressed
\[
\gamma_{lk}(s) = E \left\{ \frac{[W^{(l)}z(t)]'[W^{(k)}z(t+s)]]}{N} \right\}
\]

\[
= tr \left\{ \frac{W^{(k)'}W^{(l)}\Gamma(s)}{N} \right\},
\]

where \( tr[A] \) is the trace of the square matrix \( A \) and \( \Gamma(s) = E[z(t)z(t+s)'] \). More details, see for example Pfeifer and Deutsch (1980b) and Subba Rao and Antunes (2003).

2.3.1. Model identification

The identification of the STARMA model is carried out by using the \textit{space-time autocorrelation function} (STACF). The STACF between the \( l \) and \( k \) order neighbors, at the time lag \( s \), is defined as

\[
\rho_{lk}(s) = \frac{\gamma_{lk}(s)}{[\gamma_{ll}(0)\gamma_{kk}(0)]^{1/2}}.
\]

Given the vector \( z(t) = [z_1(t), \ldots, z_N(t)]' \) of observations at time \( t = 1, \ldots, T \), the estimator of \( \Gamma(s) \) is given by

\[
\hat{\Gamma} = \frac{T-s}{T} \sum_{t=1}^{T-s} z(t)z(t+s)', \quad s \geq 0.
\]

\( \hat{\Gamma} \) can be substituted in Equation 3 in order to obtain the sample estimates \( \hat{\gamma}_{lk} \) of the STCOV. Therefore, the sample estimator of the STACF is

\[
\hat{\rho}_{lk}(s) = \frac{\hat{\gamma}_{lk}(s)}{[\hat{\gamma}_{ll}(0)\hat{\gamma}_{kk}(0)]^{1/2}}.
\]

Pfeifer and Deutsch (1980b) demonstrated that identification can usually proceed strictly on the basis of \( \hat{\rho}_{l0} \) for \( l = 1, \ldots, \lambda \).

Each particular model of the STARMA family has a unique space-time autocorrelation function (see Table 2). However, if the model is autoregressive but with unknown order, is not easy to determine its correct order using \( \hat{\rho}_{lk}(s) \). This difficulty can be handled using the \textit{space-time partial autocorrelation function} (STPACF), which can be expressed as

\[
\rho_{0h} = \sum_{j=1}^{k} \sum_{l=0}^{\lambda} \phi_{jl} \rho_{hl}(s-j), \quad s = 1, \ldots, k; \ h = 0,1,\ldots, \lambda.
\]
(1980b), the value of $\lambda$ must be at least the maximum spatial order of any hypothetic model.

Table 2: Characteristics of the theoretical STACF and STPACF for STAR, STMA and STARMA models.

<table>
<thead>
<tr>
<th>Process</th>
<th>STACF</th>
<th>STPACF</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAR</td>
<td>Tails off with both space and time</td>
<td>Cuts off after $p$ lags in time and $\lambda_p$ lags in space</td>
</tr>
<tr>
<td>STMA</td>
<td>Cuts off after $q$ lags in time and $m_q$ lags in space</td>
<td>Tails off with both space and time</td>
</tr>
<tr>
<td>STARMA</td>
<td>Tails off</td>
<td>Tails off</td>
</tr>
</tbody>
</table>

2.3.2. Parameter estimation

Assuming that the $\epsilon(t)$, $t = 1, \ldots, T$, are independent with distinct variances for each of the $N$ sites, that is, the variance-covariance matrix $G$ is a $N \times N$ diagonal matrix, the maximum likelihood estimates of

$$
\Phi = [\phi_{10}, \ldots, \phi_{1\lambda_1}, \ldots, \phi_{p0}, \ldots, \phi_{p\lambda_p}]',
$$

$$
\Theta = [\theta_{10}, \ldots, \theta_{1\lambda_1}, \ldots, \theta_{q0}, \ldots, \theta_{qm_q}]',
$$

the parameter vectors of the STARMA model defined in Equation 1, are obtained by maximizing the log-likelihood function

$$
l(\epsilon|\Phi, \Theta, G) = -\frac{TN}{2} \log |2\pi G| - \frac{1}{2} \sum_{t=1}^{T} \epsilon(t)'G^{-1}\epsilon(t),
$$

$$
= -\frac{TN}{2} \log |2\pi G| - \frac{1}{2} S(\Phi, \Theta)
$$

where

$$
S(\Phi, \Theta) = \sum_{t=1}^{T} \epsilon(t)'G^{-1}\epsilon(t),
$$

(6)

is the weighted sum of squares of the errors and

$$
\epsilon(t) = z(t) + \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \phi_{kl} W^{(l)} z(t - k) - \sum_{k=1}^{q} \sum_{l=0}^{m_k} \theta_{kl} W^{(l)} \epsilon(t - k).
$$
Finding the values of the parameters that minimize the log-likelihood function is equivalent to finding the values \( \hat{\Phi} \) and \( \hat{\Theta} \) that minimize the sum of squares in Equation 6. Therefore, the problem is reduced to finding the weighted least squares estimates of the parameters.

Numerical techniques must be used to minimize the sum of squares in Equation 6. Subba Rao and Antunes (2003) proposed a procedure for initial estimation of the parameters of \( S(\Phi, \Theta) \) as well as an efficient criterion for order determination.

2.3.3. Model Adequacy

If the fitted model represents adequately the data, the residuals should have gaussian distribution with mean zero and variance-covariance matrix equal to \( G \). There are several tests to verify these conditions in the residuals. Particularly, Pfeifer and Deutsch (1980a) and Pfeifer and Deutsch (1981) suggested to calculate the sample space-time autocorrelations of the residuals and to compare them with their theoretical variance. The authors proved that, if the model is adequate,

\[
\text{var}(\hat{\rho}_{l0}(s)) \approx \frac{1}{N(T-s)},
\]

where \( \approx \) means approximately and \( \hat{\rho}_{l0}(s) \) is the space-time autocorrelation function of the fitted model residuals. Since the space-time autocorrelations of the residuals should be approximately gaussian, they can be standardized for, subsequently, testing their significance.

Pfeifer and Deutsch (1980a) pointed out that if the residuals have spatial correlation they can be represented by a STARMA model. Usually, identifying the model and incorporating into the candidate model that generated the residuals, is the best form of updating the model.

According to Subba Rao and Antunes (2003), the estimated parameters can be tested for statistical significance in two ways: use the confidence regions for the parameters to test the hypothesis that \( H_0 : \Phi = \Theta = 0 \), or test the hypothesis that a particular \( \phi_{kl} \) or \( \theta_{kl} \) is zero with the remaining parameters unrestricted.

Let \( \hat{\delta} = (\hat{\Phi}, \hat{\Theta})' = (\delta_1, \ldots, \delta_K)' \) be the least squares estimate of the full parameter vector, and let \( \delta^* = (\delta_1, \ldots, \delta_i, \ldots, \delta_K)' \) be the least squares estimate of the parameter vector with \( \delta_i, i = 1, \ldots, K \), constrained to be zero. The test for the hypothesis \( H_0 : \delta_i = 0 \)
is based on the statistic:

\[ \Upsilon = \frac{(TN - K)[S(\hat{\delta}^*) - S(\hat{\delta})]}{S(\hat{\delta})}. \]

Under \( H_0 \), \( \Upsilon \) is approximately distributed as an \( F_{1,TN-K} \). Any parameter that is statistically insignificant must be removed from the model to obtain a simpler model which must be considered as candidate and the estimation stage must be repeated.

3. Results and discussion

3.1. Data preparation

Outliers detection

Figure 2: SO\(_2\) daily average concentrations at the AAQMN monitoring stations (--- 2005 WHO guideline —— 2005 WHO interim guideline).

Figure 2 shows the time series plots of the six monitoring stations considered in this study. Some sites (like Laranjeiras at the beginning of the year 2009, for example) show outliers that can affect the modeling and forecasting model performance.

In this context, Fox (1972) suggested four classes of outliers: additive outliers (AO), level shift (LS), temporal change (TC) and innovational outliers (IO). According to (Peña,
the effect of AO, TC and LS outliers is limited and independent of the model, AO and TC have transitory effects while LS have permanent effects. However, the effect of an IO depends on the kind of model and its statistical characteristic.

We used the methodology proposed by Gomez and Maravall (1998), which is implemented on the software TRAMO (http://www.bde.es/), for outliers detection and correction of the time series obtained from each monitoring station. Table 3 shows the number of the observation detected as outlier as well as its type.

There were not any IO outliers and the only LS outlier was detected in Cariacica corresponding to observation 568 (July 22, 2006). This level shift can be observed in Figure 2, there is a sudden fall of concentrations observed from this date on, maybe because of a measuring equipment change or any calibration adjusting of the equipment.

Almost all time series observed have outliers with immediate effects, like observation 1536 in Laranjeiras, recorded on March 16th, 2009 (AO outlier); or short-time effects (TC outliers), like the observation 848 in Enseada do Suá, corresponding to April 28th, 2007, where there is a temporary fall in the concentrations, but rapidly they back to the mean levels.

Considering the high quantity of outliers detected by the previous analysis, we decided to transform all the time series in order to correct the distortions caused by the atypical values.

Cycles determination

It is well known that air pollution and meteorological data are influenced by cycles and seasons. In order to determine the cycles affecting SO$_2$ daily average concentrations, we estimated the periodogram for the time series from each monitoring station. The plots of the periodograms are not shown due to space constraints, however, the most significant periods are given in Table 4.

The expected period of 7 days (since the time series are daily measurements) is significant only in Vitória Centro and Cariacica stations, both sites also present significant periods of 3.5 and 32 days. The remaining monitoring stations have significant periods of approximately 19, 57 and 82 days. These findings indicate that SO$_2$ concentration levels are affected not only by weekly cycles, but also by monthly and seasonal periods. Following Antunes and Subba Rao (2006), we removed the cyclical component in each time series.
Denoting by $Y(t)$ the outliers-corrected time series, the transformed series to be used for STARMA modeling can be written as

$$Z(t) = Y(t) - X(t),$$

where $X(t) = [X_1(t), \ldots, X_6(t)]'$ is a periodic function that can be represented as a harmonic series, i.e.

$$X_i(t) = \sum_{j=1}^{s} \left[ \xi_{i,j} \cos \left( \frac{2\pi j t}{C_j} \right) + \xi_{i,j}^\dagger \sin \left( \frac{2\pi j t}{C_j} \right) \right], \quad i = 1, \ldots, 6, \quad t = 1, \ldots, T$$

where $\xi_{i,j}$ and $\xi_{i,j}^\dagger$ are unknown parameters which are estimated by least squares, $s$ is the number of significant cycles and $C_j$ represents the period (or cycle) of the time series.
Table 4: Significant cycles by monitoring station.

<table>
<thead>
<tr>
<th>Station</th>
<th>Cycle (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laranjeiras</td>
<td>None</td>
</tr>
<tr>
<td>Enseada do Suá</td>
<td>16.5, 17.5, 18.5, 82</td>
</tr>
<tr>
<td>Vitória Centro</td>
<td>32, 7, 3.5, 19</td>
</tr>
<tr>
<td>Ibes</td>
<td>18.5, 16.5, 57, 25</td>
</tr>
<tr>
<td>Vila Velha Centro</td>
<td>82, 56.5, 18.5, 75</td>
</tr>
<tr>
<td>Cariacica</td>
<td>7, 3.5, 32</td>
</tr>
</tbody>
</table>

3.2. Descriptive analysis

As observed on Figure 2, for every year the average concentrations are lower than the standard level established by the Brazilian law (CONAMA N° 03 of 28/06/90) which are: average of 365 µg/m³ for a 24-hour period (cannot be exceeded more than once a year) and annual arithmetic average of 80 µg/m³. Nevertheless, the concentrations are quite higher than the guideline suggested by the World Health Organization (World Health Organization [WHO], 2006), which is 24-hour average concentration of 20 µg/m³, or even the interim guideline of 50 µg/m³ average suggested for developing countries like Brazil.

Particularly, Vila Velha Centro station exceed the interim limit only once in 2006. Cariacica station does not exceed any limit and shows the lowest values and variability.

These assertions can be confirmed from the results displayed in Table 5. Besides, it can be observed that some stations show a high variability and maximum values much larger than the most of observed concentrations, for example, while 75% of concentrations from Ibes station is lower than 14.48 µg/m³, the maximum concentration observed is 41.385 µg/m³ (more than four times the mean value).

Table 5: Summary statistics of daily average SO₂ concentrations in GVR (2005-2009).

<table>
<thead>
<tr>
<th>Station</th>
<th>Minimum</th>
<th>1st. Quart</th>
<th>Median</th>
<th>Mean</th>
<th>3rd. Quart</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enseada do Suá</td>
<td>2.159</td>
<td>10.349</td>
<td>14.195</td>
<td>14.942</td>
<td>18.452</td>
<td>47.288</td>
</tr>
<tr>
<td>Vitória Centro</td>
<td>2.417</td>
<td>9.651</td>
<td>13.233</td>
<td>14.165</td>
<td>17.915</td>
<td>42.295</td>
</tr>
<tr>
<td>Ibes</td>
<td>0.623</td>
<td>5.738</td>
<td>9.694</td>
<td>10.898</td>
<td>14.476</td>
<td>41.385</td>
</tr>
<tr>
<td>Vila Velha Centro</td>
<td>1.288</td>
<td>8.914</td>
<td>11.195</td>
<td>12.422</td>
<td>14.918</td>
<td>54.165</td>
</tr>
<tr>
<td>Cariacica</td>
<td>0.479</td>
<td>6.316</td>
<td>7.927</td>
<td>7.872</td>
<td>9.797</td>
<td>17.852</td>
</tr>
</tbody>
</table>
The highest SO$_2$ mean concentrations were observed at Enseada do Suá and Vitória Centro stations. This situation can be explained by the direct influence of industrial and port activities for both monitoring stations, as showed in Table 1.

The boxplots shown in Figure 3 show that the mean concentrations and variability are different for all stations. Higher concentrations are observed in regions influenced by the main industrial activities of GVR, and lower values are observed in regions far away from that influence (like Laranjeiras and Cariacica stations). This behavior suggests there is an influence of the location, which reinforces the importance of including spatial characteristics into the model.

Figure 4 displays the boxplots of the average concentrations by day of the week. As observed in Section 3.1, there is a weekly cycle in Vitória Centro and Cariacica monitoring stations because the median is slightly lower on weekends and the concentration rises along the week. The remaining stations do not show any obvious trend along the week.

The sample autocorrelation functions (ACF) of the outliers-corrected SO$_2$ time series obtained for each monitoring station are shown in Figure 5. The slow decay of the correlations suggest non-stationarity of the time series in all the stations, however, the Augmented Dickey-Fuller test, proposed by Dickey and Fuller (1979), was used to examine the hypothesis of stationarity of SO$_2$ average concentrations at each monitoring station. Results indicate that there is not enough evidence to consider the series as non-stationary.
Figure 4: Boxplots of SO$_2$ daily average by day of the week.

(p-value < 0.02 for all stations).

3.3. Weighting matrix

As indicated by Pfeifer and Deutsch (1980b), the weighting matrix $W^{(l)}$ must be defined prior to modeling. Since the GVR has a small number of stations irregularly distributed over a relatively small area, it is reasonable to consider each site as first order neighbor of every other site. Therefore, the maximum spatial order of the STARMA model is one. So we have

$$W^{(0)} = I_N \quad \text{and} \quad W^{(1)} = W.$$

There are several ways to define the weighting matrix, see Cliff and Ord (1981) and Anselin and Smirnov (1996). In particular, we chose $W$ formed by weights inversely proportional to the Euclidean distance between the monitoring stations since this is the most widely used and simplest approach.

The distance (Km) between the stations was calculated using the expression:

$$d_{ij} = 6378.7 \times \cos(\text{lat}_i/57.296) \times \sin(\text{lat}_j/57.296) + \cos(\text{lat}_i/57.296) \cos(\text{lat}_j/57.296)$$

$$\times \cos(\text{lon}_j/57.296 - \text{lon}_i/57.296),$$
Figure 5: Autocorrelation Functions for SO$_2$ daily average by monitoring station.

for $i, j = 1, 2, \ldots, 6$, where lat$_i$ and lon$_i$ represent the latitude and longitude of the station $i$, respectively (www.meridianworlddata.com/Distance-Calculation.asp). Therefore, the weighting matrix $W$ was defined considering weights ($w_{ij}$) as,

$$w_{ij} = \begin{cases} 
1/d_{ij}, & \text{for } i \neq j \\
0, & \text{for } i = j.
\end{cases}$$

The weights were scaled so that the sum of the elements at each line equals one. The resulting $W$ matrix is:

$$W = \begin{bmatrix}
0.000 & 0.252 & 0.206 & 0.184 & 0.211 & 0.148 \\
0.081 & 0.000 & 0.212 & 0.211 & 0.409 & 0.087 \\
0.073 & 0.232 & 0.000 & 0.299 & 0.235 & 0.161 \\
0.058 & 0.208 & 0.269 & 0.000 & 0.348 & 0.118 \\
0.060 & 0.359 & 0.188 & 0.311 & 0.000 & 0.082 \\
0.096 & 0.176 & 0.297 & 0.242 & 0.188 & 0.000 
\end{bmatrix}$$
3.4. Fitted model

From Figures 6 and 7 we can observe that there is no remaining seasonality or cycles in the data. According to the characteristics described on Table 2, the slow decaying of the STFAC and the cutting-off in the STPACF after the first 6 time lags in the spatial lag zero indicates that a suitable model is a STAR with maximum autoregressive order 6.

The partial space-time autocorrelations are not significant for the spatial order 1 after the first time lag, indicating that a spatial order one could be enough. The STACF and STPACF were calculated based on the assumption that the errors $\varepsilon$ have a diagonal variance-covariance matrix $G$, estimated from the data.

![STACF for SO$_2$ daily average time series](image)

Figure 6: Space-time Autocorrelation Function (STACF) for SO$_2$ daily average time series.

The model with the best performance is the STAR($4_{1,0,0,0}$) with parameters (the standard errors are shown in brackets):

\[
\begin{align*}
\phi_{10} &= -0.475 (0.0109) & \phi_{11} &= -0.066 (0.0306) \\
\phi_{20} &= -0.066 (0.0121) & \phi_{21} &= 0.058 (0.0335) \\
\phi_{30} &= -0.108 (0.0121) & \phi_{31} &= -0.004 (0.0335) \\
\phi_{40} &= -0.156 (0.0109) & \phi_{41} &= -0.019 (0.0306)
\end{align*}
\]

The parameters $\phi_{21}$, $\phi_{31}$ and $\phi_{41}$ were not significant at a 5% level of significance.
Therefore, the final fitted model is:

$$
\hat{z}(t) = 0.475z(t-1) + 0.066Wz(t-1) + 0.066z(t-2) + 0.108z(t-3) + 0.156z(t-4).$$

(7)

The sample STACF of the residuals, displayed in Figure 8, shows very small autocorrelation values, suggesting that the assumption of uncorrelated errors is satisfied by the fitted model.

Normality tests and quantile-quantile plots of the residuals (Figure 9) show that the errors are not normally distributed. The lack of Gaussian distribution affects only the inferential process, that is, the significance tests as well as the confidence and prediction intervals.

In order to guarantee the reliability of the model, bootstrap resampling techniques were used to obtain confidence intervals for the estimated parameters as well as the prediction intervals. The bootstrap approach here adopted was resampling from the residuals $\varepsilon(t)$ of the fitted model as follows,

a. Calculate the residual for each observation:

$$
\hat{\varepsilon}(t) = z(t) - \hat{z}(t) \quad t = 1, \ldots, T.
$$
Figure 8: Space-time Autocorrelation Function (STACF) of the residuals from the fitted STARMA(4,0,0,0,0) model.

b. Select bootstrap samples of the residuals, $e^*_b = [e^*_b(1), \ldots, e^*_b(T)]'$, and from these, calculate bootstrapped $z$ values $z^*_b = [z^*_b(1), \ldots, z^*_b(T)]'$, where $z^*_b(t) = \tilde{z}(t) - e^*_b(t)$, for $t = 1, \ldots, T$.

c. Fit the model using $z$ values to obtain the bootstrap coefficients

$$\delta^*_b = (\phi^*_{10,b}, \phi^*_{11,b}, \phi^*_{20,b}, \phi^*_{21,b}, \phi^*_{30,b}, \phi^*_{31,b}, \phi^*_{40,b}, \phi^*_{41,b})',$$

for $b = 1, \ldots, r$, where $r$ is the number or bootstrap replicates.

d. The resampled $\delta^*_b$ can be used to construct bootstrap standard errors and confidence intervals for the coefficients.

As is well known, the bootstrap samples have the property of mimic the original sample. More details about bootstrap techniques can be obtained in Wu (1986), Efron and Tibshirani (1993) and Lam and Veall (2002) among others.

Figure 10 displays the predicted values of the observed time series by using the fitted model. This figure suggests a reasonably good performance of the model. It well captures the variability, tendency and the periods of the data.

The model indicates that SO$_2$ concentrations in a site are highly influenced by the levels presented in the previous day ($\phi_{10} = -0.475$). Moreover, the permanence of SO$_2$
in the atmosphere of the region is around 3-4 days and the concentration level in a site is influenced by the concentration observed at its neighbors in the day before. Based on the good in-sample performance of the model, it is reasonable to consider it as an alternative method for estimating missing data.

### 3.5. Forecasting

The fitted model shown in Equation 7 was used in order to determine one-step-ahead forecasts for a 15-days period, that is, we obtained forecasts for the last two weeks of the full period. The forecasts were calculated using the Minimum Mean Square Error (MMSE) criterion as

\[
\hat{z}_{(1)}(t) = E[z(t + 1)|z(s), s \leq t].
\]

The forecasts and their 95% prediction intervals are displayed in Figure 11. It can be observed that forecasts describe well the time series behavior and trend for all the stations. Even knowing that Gaussian distribution assumption is not met, the prediction intervals under this supposition were calculated only for comparative purposes. It becomes clear that the errors were underestimated for the most of stations and, therefore, the reliability of the inferences based on the Gaussian assumption was strongly compromised. This fact reinforces the usefulness of the resampling techniques in order to perform efficient
inferences.

In particular, for the time series which have the lower variability (Laranjeiras and Cariacica stations), almost all the real data falls within the prediction intervals and their forecasts are more accurate than those for the sites which have observations very distant from the mean, as is the case of Enseada do Suá station, for example. For the remaining series, it can be observed that even the model capturing the high variability in the data, the discrepant values are not covered by the prediction intervals.

In order to quantify the forecasting ability of the fitted model for each monitoring station we used the criterions: root mean squared error (RMSE) and mean absolute error (MAE), defined as

\[ RMSE_i = \sqrt{\frac{1}{H} \sum_{t=T+1}^{T+H} \epsilon_i(t)^2}, \]

\[ MAE_i = \frac{1}{H} \sum_{t=T+1}^{T+H} |\epsilon_i(t)|, \]

where \( i = 1,2,\ldots,6 \) and \( H = 1,\ldots,15 \). The MAE measures the average magnitude of errors considering their absolute magnitude. The RMSE is also known as the standard
error of the forecast and it is more sensitive to outliers than MAE (Hyndman and Koehler, 2006).

As observed in Table 6, Laranjeiras and Cariacica stations have the most accurate forecasts (MAE of about 1.71 and 0.25, respectively). The highest values for the MAE criterion were obtained for Ibes, Enseada do Suá and Vitória Centro stations (about 2.64, 2.59 and 2.11, respectively), which means that the average absolute difference between the forecasts and the observed concentrations was approximately 2 $\mu$g/m$^3$.

The most imprecise forecasts were obtained for Enseada do Suá with a residual standard deviation of 3.04 $\mu$g/m$^3$, followed by Ibes station which has a RMSE of 2.91 $\mu$g/m$^3$.

4. Final Remarks

This study applies a STARMA model to daily average SO$_2$ concentrations in order to describe the dynamics of the pollutant at GVR, as well as to forecast future concentrations. The analysis of the individual time series at the monitoring stations reveals that there are some significant cycles affecting the behavior of the dispersion over the region.
Based on the fitted model, the persistence of SO$_2$ in the region is about four days and its concentration levels are influenced by the levels observed at nearby sites. The residual analysis indicated a good fit for in-sample observations, so that it can be used for imputation of missing values. Regarding the out-of-sample performance, the model can be a reasonable tool for predicting future values with a certain reliability. The higher values of the accuracy measures for the series with more discrepant values indicate that the forecasting capability of the model is highly influenced by outliers.

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