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Remarks on Pricing Asian Power Options under

Mixed Fractional Brownian Motion Environment

with Superimposed Jumps

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Abstract : It has been observed that the stock price process can be modeled with driving force as a mixed fractional Brownian motion with Hurst index $H > \frac{3}{4}$ whenever long-range dependence is possibly present. We propose a geometric mixed fractional Brownian motion model for the stock price process with possible jumps superimposed by an independent Poisson process. We remark on computation of the price of a geometric Asian option under such an environment.

1 Introduction

Estimation of option price is an important problem in mathematical finance. A call option is a contract which gives the holder the right but not obligation to buy a risky asset at a certain date called the strike date or the exercise date with a predetermined price called the strike price or the exercise price. A put option is a contract which gives the holder the right but not obligation to sell a risky asset at a certain date called the strike date or the exercise date with a predetermined price called the strike price or the exercise price. There are several types of options that are traded in a market. American option allows the owner to exercise his option at any time up to and including the strike date. Bermuda options permit the owner to exercise his option early but only on a contractually specified finite set of dates. European options can be exercised only on the strike date. European options are also called vanilla options. Their payoffs at maturity depend on the spot value of the stock at the time of exercise. There are other options whose value depends on stock prices over a

predetermined time interval. For an Asian option, the payoff is determined by the average value over some predetermined time interval. Asian options reduce the volatility inherent in the option and are cheaper compared with European option (cf. Mao and Liang (2014), Prakasa Rao (2013)). For modelling of fluctuations in movement of stock prices, Brownian motion has been used traditionally as the driving force for modelling log returns. It has been noted later that there might be long-range dependence in the phenomena and the log returns have possibly heavy tailed distibutions. It was suggested by some that the driving force for modelling of price movement may be chosen as a fractional Brownian motion. Bjork and Hult (2005) and Kuznetsov (1999) observed that the use of fractional Brownian motion for modelling fluctuations in movement of stock prices is not justifiable as it allows arbitrage opportunities. To avoid this problem, Cherdito (2000, 2003) suggested the use of a mixed fractional Brownian motion as a suitable model to capture the fluctuations of the financial assets. The mixed fractional Brownian motion (mfBm) is a Gaussian process that is a linear combination of the Brownian motion and a fractional Brownian motion with Hurst index $H > 1/2$. Cheridito (2001) has proved that, for $H \in (3/4, 1)$, the mfBm is equivalent to a Brownian motion and hence modeling price fluctuation via mfBm allows arbitrage-free market. Xiao et al. (2012) studied pricing model for equity warrants in a mixed fractional Brownian environment. Sun (2013) investigated pricing currency options when the driving force is a mixed fractional Brownian motion. Yu and Yan (2008) discussed European call option pricing under a mixed fractional Brownian motion environment. Mao and Liang (2014) evaluated a geometric Asian option price under a fractional Brownian motion frame work. They derived a closed form for the solution for the Asian power option price. The pricing of currency options in a mixed fractional Brownian motion with a jump environment has been studied in Foad and Adem (2014) and Prakasa Rao (2015). Sun and Yan (2012) discussed use of mixed-fractional models in credit risk pricing. Our aim is to investigate computing the price of Asian option under a mixed fractional Brownian motion environment.

2 Asian Options

The payoff of an Asian option is determined by the average value of the stock price over a pre-fixed time interval. As has been pointed out by Mao and Liang (2014), they reduce the risk of market manipulation of the underlying instrument at maturity and reduce the volatility in the option. Furthermore, Asian options are generally cheaper than the corresponding European options. Asian options are of different types such as fixed strike price options and floating

strike price options. The payoff for a fixed strike price option is $(A(T)-K)_+$ and $(K-A(T))_+$ for a call and put option respectively where K denotes the strike price, T is the strike time and $A(T)$ is the average price of the underlying asset over the predetermined interval. For a floating strike price option, the payoffs are $(S(T) - A(T))_+$ and $(A(T) - S(T))_+$, for a call and put option respectively where $S(T)$ is the price of stock at time T. Asian options can again be differentiated in to two classes: one is the arithmatic average, that is,

$$A(T) = \frac{1}{T} \int_0^T S(t)dt$$

and the other is the geometric average

$$A(T) = \exp\{\frac{1}{T} \int_0^T \log S(t)dt\}$$

assuming that the pre-fixed interval for computing the average is the interval $[0, T]$. We will consider evaluation of Asian option price in the case of continuous geometric average with a fixed strike price in an mfBm environment.

3   Mixed fractional Brownian Motion

We now define the mixed fractional Brownian motion (mfBm) and discuss some of its properties.

A mixed fractional Brownian motion $M^H(\alpha, \beta)$ is a linear combination of a Brownian motion and a fractional Brownian motion (fBM) with Hurst index H, that is,

(3. 1) $$M_t^H(\alpha, \beta) = \alpha W_t + \beta W_t^H, 0 \leq t < \infty$$

where W is the standard Brownian motion and W^H is an independent standard fractional Brownian motion with Hurst index H and α, β are some real constants not both zero. The equality here is understood in the sense that the finite dimensional distributions of the process on the left side of the equation (3.1) are the same as the corresponding finite dimensional distributions of the process on the right side of the equation (3.1). The process $M^H(\alpha, \beta)$ is a centered Gaussian process with $M_0^H = 0$ a.s.and with the covariance function

$$cov(M_t^H, M_s^H) = \alpha^2 \min(t, s) + \frac{\beta^2}{2}(t^{2H} + s^{2H} - |t - s|^{2H}).$$

The increments of the process $M^H(\alpha, \beta)$ are stationary and self-similar, in the sense, for any $h > 0$,

(3. 2) $$M_{ht}(\alpha, \beta) \triangleq M_t^H(\alpha h^{1/2}, \beta h^H).$$

Here Δ indicates that the random variables on both sides of the equation (3.2) have the same distribution. The increments of the process are positively correlated if $\frac{1}{2} < H < 1$, uncorrelated if $H = \frac{1}{2}$ and negatively correlated if $0 < H < \frac{1}{2}$. The increments of the process are long-range dependent if and only if $\frac{1}{2} < H < 1$. For more details on the properties of a mfBm, see Zili (2006) and Prakasa Rao (2010). We assume here after that that the index $H > \frac{3}{4}$ which ensures the fact the probability measure generated by the process $M^H(\alpha, \beta)$ is equivalent to the Wiener measure. For simplicity in computations, we assume that $\alpha = \beta = 1$ here after.

4 Pricing model when the driving force is an mfBm

We assume that the following assumptions hold: (i) the dynamics of the underlying stock price follows the mixed fractional Brownian motion (mfBM) with Hurst index $H > \frac{3}{4}$; (ii) the risk-free interest rate r and the dividend rate q are constant; (iii) there are no transaction costs in buying or selling the stocks or options, that is, the market is frictionless; and (iv) the option can be exercised only at the time of maturity.

The fact that the Hurst index of the mfBm is $H > \frac{3}{4}$ ensures that the market does not admit arbitrage opportunity. Suppose the stock price S_t at time t satisfies the stochastic differential equation (SDE)

(4. 1) $\qquad dS_t = \mu S_t dt + \sigma S_t d\bar{W}_t + \sigma S_t d\bar{W}_t^H, S_0 = S(0) > 0, 0 \le t \le T$

where $\bar{W}$ is the standard Brownian motion and $\bar{W}^H$ is an independent standard fBm with the Hurst index $H > \frac{3}{4}$. The dynamics of the stock price process $\{S(t), t \ge 0\}$ will satisfy the SDE

(4. 2) $\qquad dS_t = (r - q)S_t dt + \sigma S_t dW_t + \sigma S_t dW_t^H, S_0 = S(0) > 0, 0 \le t \le T$

under the risk neutral probability measure from the general theory of option pricing and the fact that the risk-neutral measure is a martingale measure (cf Shreve (2004); Cheridito (2003)). Here W is a standard Brownian motion and W^H is an independent standard fBm with the Hurst index $H > \frac{3}{4}$. The solution of the SDE (4.2) is

(4. 3) $\qquad S_t = S_0((r - q)t + \sigma(W_t + W_t^H) - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}), 0 \le t \le T.$

This implies that the stock price $S(t)$ is log-normally distributed with

$$\log S(t) \simeq N(\log S(0) + (r - q)t - \frac{1}{2}\sigma^2 t - \frac{1}{2}\sigma^2 t^{2H}, \sigma^2 t + \sigma^2 t^{2H}).$$

Here $N(m, \nu^2)$ denotes the Gaussian distribution with mean m and variance ν^2. Let $C(S(0), T)$ be the price of an European call option at time 0 with strike price K that matures at time T. Following Theorem 4.1 in Sun (2013) (cf. Yu and Yau (2008)), it follows that

(4. 4)
$$C(S(0), T) = S(0)e^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

where

$$d_1 = \frac{\log(S(0)/K) + (r - q)T + \frac{\sigma^2}{2}T + \frac{\sigma^2}{2}T^{2H}}{\sqrt{\sigma^2 T + \sigma^2 T^{2H}}}$$
$$d_2 = d_1 - \sqrt{\sigma^2 T + \sigma^2 T^{2H}},$$

and $\Phi(.)$ denotes the standard normal distribution function. Applying the put-call option parity formula (cf. Ross (2003), Prakasa Rao (2013)), it is easy to obtain the option price for an European put option under the above scenario.

5   Adding Jumps to Geometric Mixed Fractional Brownian Motion

It is now known that modeling of the stock price process using the geometric Brownian motion may not be useful in general as it does now allow possibility of a discontinuous price jump either in the upward or the downward direction and is not suitable to model long range dependence if present. Under the assumption of the geometric Brownian motion, the probability of having a jump is zero. Since such jumps do occur in practice for various reasons, it is important to consider a model for the stock price process that allows possibility of jumps in the process.

Suppose there are no transaction costs, trading is continuous and the interest rate r is constant and compounded continuously. We have indicated that there are no arbitrage opportunities under the mixed fractional Brownian motion model whenever the Hurst index $H \in (\frac{3}{4}, 1]$.

We now propose a model for the stock price process which is a jump mixed fractional Brownian motion model to capture the jumps or the discontinuities and the fluctuations in the stock price process and to take into account the long range dependence of the stock price process if present. We obtained the European call option price for such models in Prakasa Rao (2015a) (cf. Foad and Adem (2014)).

Fractional Brownian motion with superimposed jumps can be used for pricing currency options (cf. Xiao et al. (2010)). It is based on the assumption that the exchange rate returns are generated by a two-part stochastic process, the first part dealing with small continuous price movements generated by a mixed fractional Brownian motion and the second part by large infrequent price jumps generated by a Poisson process. As has been pointed out by Foad and Adem (2014), modeling, by this two-part process, is in tune with the market in which major information arrives infrequently and randomly. In addition, this process provides a model through heavy tailed distributions for modeling empirically observed distributions of exchange rate changes. We suggest a mixed fractional Brownian motion with superimposed jumps for modelling the stock price process or for modeling currency options.

We now introduce the Poisson process as a model for the jump times in the stock price process. Let $N(0) = 0$ and let $N(t)$ denote the number of jumps in the process that occur by time t for $t > 0$. Suppose that the process $\{N(t), t \geq 0\}$ is a Poisson process with stationary independent increments. Under such a process, the probability that there is a jump in a time interval of length h is approximately λh for h small and the probability of more than one jump in a time interval of length h is almost zero for h sufficiently small. Furthermore the probability that there is a jump in an interval does not depend on the information about the earlier jumps. Suppose that, when the i-th jump occurs, the price of the stock is multiplied by an amount J_i and the random sequence $\{J_i, i \geq 1\}$ forms an independent and identically distributed (i.i.d.) sequence of random variables. In addition, suppose that the random sequence $\{J_i, i \geq 1\}$ is independent of the times at which the jumps occur. Let $S(t)$ denote the stock price at time t for $t \geq 0$. Then

$$S(t) = S^*(t)\Pi_{i=1}^{N(t)} J_i, t \geq 0$$

where $\{S^*(t), t \geq 0\}$ is the geometric mixed fractional Brownian motion modeled according to the stochastic differential equation

$$dS_t^* = \mu S_t^* dt + \sigma S_t^* d\bar{W}_t + \sigma S_t^* d\bar{W}_t^H, S_0^* = S(0), 0 \leq t \leq T.$$

We assume that the process $\{S^*(t), t \geq 0\}$ and the Poisson process $\{N(t), t geq 0\}$ are *independent*. Note that, if there is a jump in the price process at time t, then the jump is of size J_i at the i-th jump. Let

$$J(t) = \Pi_{i=1}^{N(t)} J_i, t \geq 0$$

and we define $\Pi_{i=1}^{N(t)} J_i = 1$ if $N(t) = 0$. Note that the random variable $\log \frac{S^*(t)}{S^*(0)}$ has the Gaussian distribution with mean μt and variance $\sigma^2 t + \sigma^2 t^{2H}$. Note that $S(0) = S^*(0)$ is the

initial stock price and we assume that it is non-random. Observe that

$$E[S(t)] = E[S^*(t)J(t)] = E[S^*(t)]E[J(t)]$$

by the independence of the random variables $S^*(t)$ and $J(t)$. Furthermore

(5. 1)
$$\begin{aligned} E[S^*(t)] &= S^*(0)E[\exp\{\mu t + \sigma W_t^H + \sigma W_t\}] \\ &= S^*(0)\exp\{\mu t + \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t\} \end{aligned}$$

by the independence of the processes $\{W_t^H, t \geq 0\}$ and $\{W_t, t \geq 0\}$ and the properties of Gaussian random variables. It is easy to check that

$$E[J(t)] = e^{-\lambda t(1 - E[J_1])}$$

and

$$Var[J(t)] = e^{-\lambda t(1 - E[J_1^2])} - e^{-2\lambda t(1 - E[J_1])}.$$

In particular, the equations given above show that

$$E[S(t)] = S^*(0)e^{\mu t + \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t}e^{-\lambda t(1 - E[J_1])}.$$

Suppose the interest rate r is compounded continuously. Then the future value of the stock price $S(0)$, after time t, should be $S(0)e^{rt}$ under any risk-neutral probability measure. Under the no arbitrage assumption, it follows that

$$S^{(*)}(0)\exp\{\mu t + \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t - \lambda t(1 - E[J_1])\} = S(0)e^{rt}.$$

Since $S(0) = S^*(0)$, it follows that the stock price process should satisfy the relation

$$\mu t + \frac{1}{2}\sigma^2 t^{2H} + \frac{1}{2}\sigma^2 t - \lambda t(1 - E[J_1]) = rt$$

under the risk-neutral probability measure, which implies that

$$\mu t = rt - \frac{1}{2}\sigma^2 t^{2H} - \frac{1}{2}\sigma^2 t + \lambda t(1 - E[J_1])$$

under the no arbitrage assumption. The price for an European call option with strike price K, strike time t, and the interest rate r compounded continuously is equal to

$$E[e^{-rt}(S(t) - K)_+]$$

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where the expectation is computed with respect to the Gaussian distribution with the mean

$$rt - \frac{1}{2}\sigma^2 t^{2H} - \frac{1}{2}\sigma^2 t + \lambda t(1 - E[J_1])$$

and the variance

$$\sigma^2 t^{2H} + \sigma^2 t.$$

Here $a_+ = a$ if $a \geq 0$ and $a_+ = 0$ if $a < 0$. Let R_t be a Gaussian random variable with mean $rt - \frac{1}{2}\sigma^2 t^{2H} - \frac{1}{2}\sigma^2 t + \lambda t(1 - E[J_1])$ and variance $\sigma^2 t^{2H} + \sigma^2 t$. Note that the option price for the European call option under this model is

$$(5.\ 2) \qquad \begin{aligned} E[e^{-rt}(S(t) - K)_+] &= e^{-rt} E[(J(t)S^*(t) - K)_+] \\ &= e^{-rt} E[(J(t)S^*(0)e^{R_t} - K)_+] \end{aligned}$$

where $S^*(0) = S(0)$ is the initial price of the stock.

We have obtained closed form expression for option price for an European call option when the stock price process is driven by a geometric mixed fractional Brownian motion with super imposed jumps by an independent Poisson process in Prakasa Rao (2015a).

6 Pricing for Asian options when the interest and dividend rates are constant and jumps are present under mfBm environment

We have obtained a closed form expression in Prakasa Rao (2015b) for the price for the geometric Asian call option with fixed strike price K and maturity time T. when there are no jumps present and the driving force is an mfBm..

Theorem 6.1: Suppose the stock price $S(t)$ follows the model given by the SDE

$$dS_t = (r - q)S_t dt + \sigma S_t dW_t + \sigma S_t dW_T^H, S_0 = S(0) > 0, 0 \leq t \leq T$$

where W and W^H are independent Brownian motion and fractional Brownian motion with Hurst index $H > \frac{3}{4}$ under the risk-neutral probability measure and the interest rate r and the dividend rate q are constant over time. Then the price of geometric Asian call option

$C(S(0), T)$ is given by

(6. 1)
$$
\begin{aligned}
C(S(0), T) &= S(0) \exp\{-\frac{1}{2}(r+q)T - \frac{1}{12}\sigma^2 T - \frac{1}{4(2H+1)(H+1)}\sigma^2 T^{2H}\}\Phi(d_1) \\
&\quad - Ke^{-rT}\Phi(d_2)
\end{aligned}
$$

where

(6. 2)
$$
d_2 = \frac{\log(S(0)/K) + \frac{1}{2}(r-q)T - \frac{1}{4}\sigma^2 T - \frac{1}{2(2H+1)}\sigma^2 T^{2H}}{\sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}}
$$

and

(6. 3)
$$
d_1 = d_2 + \sqrt{\frac{\sigma^2 T}{3} + \frac{\sigma^2 T^{2H}}{2(H+1)}}.
$$

We now compute the price for the geometric Asian call option with the fixed strike price K and the maturity time T when the interest and dividend rates are constant and there are jumps in the stock price process as described earlier following an independent Poisson process $\{N(t), t \geq 0\}$ with intensity rate λ. Following the notation given in Section 5, let

$$
G(T) = \frac{1}{T}\int_0^T \log S(t)dt = \frac{1}{T}\int_0^T [\log S^*(t) + \log J(t)]dt
$$

and

$$
A(T) = \exp(G(T)).
$$

Suppose that $E(\log J_1) = \gamma$ and $Var(\log J_1) = \eta^2$. We will now compute the mean and the variance of the random variable $G(T)$ under the risk-neutral probability measure. Let $\tilde{E}$ denote the expectation and, $\tilde{\mu}$ and $\tilde{\sigma}^2$ denote the mean and the variance of the random variable $G(T)$ under the the risk-neutral probability measure. It is easy to check that $E(\log J(t)) = \gamma\lambda t$. Note that

(6. 4)
$$
\begin{aligned}
\tilde{\mu} &= \tilde{E}[G(T)] = \frac{1}{T}\int_0^T \tilde{E}[\log S^*(t)] + \frac{1}{T}\int_0^T \tilde{E}[\log J(t)]\, dt \\
&= \log S(0) + \frac{1}{T}\int_0^T (r-q)t dt - \frac{1}{2T}\int_0^T [\sigma^2 t + \sigma^2 t^{2H}]dt + \frac{1}{T}\int_0^T \gamma\lambda t\, dt \\
&= \log S(0) + \frac{1}{2}(r-q)T - \frac{1}{2}(\frac{\sigma^2 T}{2} + \frac{\sigma^2 T^{2H}}{2H+1}) + \frac{1}{2}\gamma\lambda T
\end{aligned}
$$

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and

(6. 5)

$$
\begin{aligned}
\tilde{\sigma}^2 &= Var[G(T)] = \tilde{E}[G(T) - \tilde{\mu}]^2 \\
&= \frac{1}{T^2} \int_0^T \int_0^T \sigma^2 (\tilde{E}[W(t)W(\tau)] + E[W^H(t)W^H(\tau)]) \, dtd\tau \\
&\quad + \frac{1}{T^2} \int_0^T \int_0^T Cov(\sum_{i=1}^{N(t)} \log J_i, \sum_{i=1}^{N(\tau)} \log J_i) dtd\tau \\
&\qquad \text{(by the independence of the processes } W \text{ and } W^H \text{ and } N \text{)} \\
&= \frac{1}{2T^2} \int_0^T \int_0^T \sigma^2 [(|t| + |\tau| - |t - \tau|) + (|t|^{2H} + |\tau|^{2H} - |t - \tau|^{2H})] \, dtd\tau \\
&\quad + \frac{1}{T^2} \int_0^T \int_0^T Cov(\sum_{i=1}^{N(t)} \log J_i, \sum_{i=1}^{N(\tau)} \log J_i) dtd\tau \\
&= \frac{1}{3} \sigma^2 T + \frac{1}{2(H+1)} \sigma^2 T^{2H} + \frac{1}{T^2} \int_0^T \int_0^T Cov(\sum_{i=1}^{N(t)} \log J_i, \sum_{i=1}^{N(\tau)} \log J_i) dtd\tau.
\end{aligned}
$$

Observing that $\{\log J_i, i \geq 1\}$ are independent and identically distributed random variables with mean γ and variance η^2 and the fact that the process $\{N(t), t \geq 0\}$ is a Poisson process with independent stationary increments, it can checked that

$$
\int_0^T \int_0^T Cov(\sum_{i=1}^{N(t)} \log J_i, \sum_{i=1}^{N(\tau)} \log J_i) dtd\tau = (\gamma^2 + \eta^2)\lambda \frac{T^3}{3}
$$

using the result

$$
Cov(g(X, Y, Z), h(X, Y, Z))
$$
$$
= Cov_Z(E(g(X, Y, Z)|Z), E(h(X, Y, Z)|Z)) + E_Z(Cov(g(X, Y, Z)|Z, h(X, Y, Z)|Z))
$$

and the elementary computation

$$
\int_0^T \int_0^T \min(t, \tau) dtd\tau = \frac{T^3}{3}.
$$

Hence

(6. 6)
$$
\tilde{\sigma}^2 = \frac{1}{3} \sigma^2 T + \frac{1}{2(H+1)} \sigma^2 T^{2H} + \frac{1}{3}(\gamma^2 + \eta^2)\lambda T.
$$

and the random variable $\log A(T)$ has the mean $\tilde{\mu}$ and the variance $\tilde{\sigma}^2$ as obtained above. However the distribution of the random variable $A(T)$ is difficult to compute explicitly in

this case and hence the corresponding for the Asian option price does not have a closed form. We now consider a special case when the random variables J_i are independent and identically distributed with log-normal distribution and try to obtain a closed form for the Asian option price.

7 Special case

Let us consider a special case of the model for the stock price process as discussed earlier. Suppose that the jumps $\{J_i, i \geq 1\}$ are i.i.d. log-normally distributed with parameters γ and η^2. It is easy to see that

$$E[J_1] = e^{\gamma + \frac{1}{2}\eta^2}.$$

Let $X_i = \log J_i, i \geq 1$. Then the random variables $X_i, i \geq 1$ are i.i.d. with Gaussian distribution with mean γ and variance η^2. Observe that

$$J(t) = e^{\sum_{i=1}^{N(t)} X_i}$$

in this special case. We now try to compute the price for the geometric Asian call option with fixed strike price K and maturity time T when the interest r and the dividend rate q are constant and there are jumps in the stock price process as described earlier following an independent Poisson process $\{N(t), t \geq 0\}$ with intensity rate λ.

Let

$$G(T) = \frac{1}{T} \int_0^T \log S(t) dt = \frac{1}{T} \int_0^T [\log S(t) + \log J(t)] dt$$

as defined earlier and

$$A(T) = \exp(G(T)).$$

We will now compute the mean and variance of the random variable $G(T)$ under the risk-neutral probability measure. Let $\tilde{E}$ denote the expectation and, $\tilde{\mu}$ and $\tilde{\sigma}^2$ denote the mean and the variance of the random variable $G(T)$ under the the risk-neutral probability measure. It is easy to check that $E(\log J(t)) = \gamma \lambda t$. The computations made in the previous section show that

(7. 1) $$\tilde{\mu} = \log S(0) + \frac{1}{2}(r - q)T - \frac{1}{2}\left(\frac{\sigma^2 T}{2} + \frac{\sigma^2 T^{2H}}{2H + 1}\right) + \frac{1}{2}\gamma \lambda T.$$

and

(7. 2) $$\tilde{\sigma}^2 = \frac{1}{3}\sigma^2 T + \frac{1}{2(H + 1)}\sigma^2 T^{2H} + \frac{1}{3}(\gamma^2 + \eta^2)\lambda T$$

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Hence the random variable $\log A(T)$ has the mean $\tilde{\mu}$ and the variance $\tilde{\sigma}^2$ as obtained above. However the distribution of the random variable $\log A(T)$ cannot be explicitly computed even in this special case as it involves sum of a Poisson number of independent identically distributed Gaussian random variables with mean γ and variance η^2. Note that the price of a geometric Asian call option is

$$C(S(0), T) = e^{-rT}\tilde{E}[(A(T) - K)_+].$$

Since the distribution of $A(T)$ can not be explicitly computed, one can possibly simulate the distribution of $A(T)$ using the definition of the random variable $A(T)$ and obtain an approximation to the option price.

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