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On the history of certain early key concepts in Sampling Theory and Practice

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Summary.

In this paper, we have outlined the historical aspects of several early innovative concepts in the theory and practice of sample surveys. These concepts relate to utilisation of auxiliary information, stratification, assessment of non-sampling errors, unified sampling theory among others. On certain occasions, researches have not paid attention to the chronology or priority of these concepts. The aim of this paper, is to place on record these aspects of some of the innovative advances that were made independently by different authors and give credit where it is due. For completeness, we also present in one place some of the scattered material in the literature on these aspects.

Key words: Basic sampling designs; Stratified sampling; Optimum allocation; Utilization of auxiliary information; Unified sampling theory; Non sampling errors.

1 Introduction

There are several early innovative concepts in the theory and practice of sample surveys which were introduced much earlier chronologically than what was taught in graduate courses or quoted in published literature. One of the aims of this paper is to trace the history of some of these concepts and bring it to the attention of researchers and teachers. While doing so, we also present in one place some of the scattered material in the literature on these topics for the sake of completeness and easy reference. We assume a basic knowledge of sampling theory and where necessary the results will be annotated. It may be noted that this is a historical review of some early concepts and our aim here is not to review the recent advances in sampling theory and practice.

Section 2 of the paper begins with the early period. Section 3 first discusses an early occasion of sampling practice during the thirties in India prior to Mahalanobis era and then moves on to mention Neyman's fundamental contributions of the thirties one of which also resulted in a

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question of priority on the aspect of allocation of sample size in stratified sampling. We note certain early developments prior to the fifties also in this section, while in section 4, a very quick survey of post fifties is mentioned. Research work done independently on the topics of basic sampling designs much earlier than what has been noted by researchers is reviewed in section 5 and antecedents to data analysis from complex surveys and small domain estimation are given in section 6. In section 7, we shall discuss the historical details of the early contributions of various authors with respect to utilisation of auxiliary information in sample surveys. Sections 8 and 9 again deal with historical issues relating to certain concepts of unified sampling theory and non sampling errors.

2 Early Period

The housing survey of 36 clusters of streets in Copenhagen, early surveys of agriculture reported from Russia, partial investigation of departments of labour and agriculture in the US and Bureau of Statistics of Canada or the surveys of British India do not really qualify to be 'representative' in the sense of those conducted by Kiaer during 1895-1903. Kiaer was the founder Director of the Norwegian Bureau of Statistics and perhaps was the first to recognise that a representative sample such as a stratified one would be time saving, less costly and yields good results with a better scope. With his regular participation and forceful presentations in the meetings of the International Statistical Institute and his experience on surveys of income distribution and living conditions in Norway, he could present empirical evidence to convince the delegates about the superiority of representative method over a complete enumeration. It was only after the 1925 Rome meeting, the report of the study commissioned by the Institute (Jessen, 1926) recommended the use of March's randomization method as well as a carefully planned purposive sampling method. Bowley, who by then had the practical experience of a successfully conducted Reading survey and who had the theoretical knowledge of the Bayesian version of Central Limit Theorem, published his work (Bowley, 1926) on inference from sample surveys. Bowley also was the first to propose selection with equal proportion of units at random from each stratum to achieve more efficient results.

3 Certain early attempts of sampling practice and theory mostly in the thirties and forties

Just as Nehru, the first Prime Minister of independent India desired that a sample survey should be organised to collect essential information for policy making and encouraged Mahalanobis to carry out such a survey, it was during the British Raj that Mahatma Gandhi desired to have statistical data collected for the Indian economy, especially the rural, and asked Kumarappa, his close associate to undertake an economic survey of Matar Taluka of Kaira district in the state of Gujarat in India which was famine stricken as an eye opener for the British rulers.

A committee consisting of Vallabh Bhai Patel as Chairman, Kumarappa as Director and a team from Gujarat Vidyapeeth, an educational institution, organised a rural survey of 54 villages during 1929 -30. The 'carefully worded and lucid report' was released in 1931 which stressed on, among other actions, reduction of taxes by the empire, tax free distribution of waste lands to the villagers for cultivation, provision of healthy seeds, regular water flow for irrigation, supply of better agricultural implements. The report comprised analysis of survey data on subjects such as crops and cultivation, manure, irrigation, marketing, industries, finances, land revenue, indebtedness, hygiene and health, cattle count, followed by recommendations (for full details, see Kumarappa(1931) and Lindley(2007)). It is interesting to observe that most of these subjects are now covered in the surveys of Indian National Sample Survey(NSS) which was established in 1950 by Mahalanobis.

The subject matter of the Gujarat survey led to the establishment of National Wastelands Development Board much later in 1985 which is now called as Department of Land Resources attached to the Indian Ministry of Rural Development.

The surveys of the NSS which started two decades after Kumarappa's Matar Taluka economic survey were nationwide and under the guidance of Mahalanobis became more sophisticated (for details see Rao(2012)).

The earliest recorded reference to Mahalanobis's interest in Sample Surveys is his lecture titled 'Elements of the Theory of Sampling' delivered on 8 July 1932 at the statistical laboratory of Presidency College, Calcutta. He defines 'sample' as follows(cf. Murthy(1963)):

“A sample consists of two or more elementary units drawn from a population (universe or field) in a random manner through the sample frame and the inference about the population is based on the observations, measurements and experiments on the variates(or characteristics) of the elementary units in the sample”.

Influenced by Hubback's work on crop cutting experiments, Mahalanobis used random sample cuts for the estimation of area under crop and crop yield. As early as in 1934, on the initiative of H.P.V. Townend, the then Development Commissioner, Bengal, he used the '*first area sample in the whole world*' for Jute forecast (Mahalanobis(1950)). During the late thirties and early forties, he introduced several innovative techniques in the surveys of the Indian Statistical Institute and presented them in the Royal Statistical Society paper(Mahalanobis(1946)) which earned him an FRS. While Mahalanobis got involved in 'Experimental Crop Census of 1937' in India, Hansen at the US Bureau contributed to the '1937 Enumerative Check Census' on Unemployment and designed a sample survey to estimate unemployment. The report by Dedrick and Hansen (1938) was treated as a '*major innovation in the philosophy of sampling*'(Hansen,1987) .

According to Deming(1973), "*.....the developments of sampling in India and in the Census in Washington followed almost parallel routes, in spite of little communication....*".

During the same decade, subsequent to the developments of twenties and based on his own research of twenties and early thirties, Neyman gave a sound basis for a random sampling technique. Neyman's influential paper of 1934, on the two different aspects of the representative method, namely the method of stratified sampling *versus* the method of purposive selection, which was presented and discussed at the Royal Statistical Society meeting introduced a variety of theoretical concepts. He gave a sound basis for a random sampling technique, invoked the concept of confidence intervals and rejected the purposive (balanced) sampling. Probability sampling thus gained momentum simultaneously in different parts of the world especially at Rothamsted Experimental Station, Iowa State College, US Bureau of Census and in India under the guidance of Mahalanobis. It may be pointed out that Hogg(1931,1932) criticized judgement sampling in favour of the use of random sampling for early employment- unemployment surveys in the U.S..

Besides several innovative ideas, Neyman's paper had the derivation of optimum allocation of sample size to strata for stratified simple random sampling (SRS) that minimises the variance of the estimate of the population mean , given by

$$n_i^{N-opt.} \propto N_i S_i$$

where N_i and S_i are respectively the size and within stratum standard deviation of the i th. stratum when samples are drawn from each stratum by SRS without replacement scheme (Neyman,1934).

It is known that Tschuprow(1923) also derived this allocation and the question of order of priority surfaced . For full details we refer to Fienberg and Tanur (1995,1996) who gave all chronological details. In view of the following acknowledgement of priority by Neyman's(1952):

“ I am obliged to Dr. Donovan J. Thompson of the Statistical Laboratory, Iowa State College, Ames, Iowa, for calling my attention to the article of A.A. Tschuprow, ‘On the mathematical expectation of the moments of frequency distributions in the case of correlated observations’ published in Metrika, Vol.2, No.4(1923), pp.646-680, which contains some results refound by me and published, without reference to Tschuprow, in 1933. The results in question are the general formula for the variance of the estimate of a mean in stratified sampling and the formula determining the optimum stratification of the sample. These formulae appeared first in a Polish booklet ‘An Outline of the Theory and Practice of Representative Method, Applied in Social Research’ published in 1933 by the Warsaw Institute of Social Problems. Later on they were republished in English in the Journal of Royal Statistical Society, Vol.97 (1934), pp.558-625. Finally, the same formulae, again without a reference to Professor Tschuprow, were given in the second edition of my book, Lectures and Conferences on Mathematical Statistics and Probability, Washington, D.C., 1952. The purpose of this note is, then, to recognize the priority of Professor Tschuprow, to express my regret for overlooking his results and to thank Dr. Thompson for calling my attention to the oversight”,

the so called ‘Neyman allocation’ as taught in graduate courses may now be referred to as ‘Tschuprow -- Neyman allocation’:

$$n_i^{TN-opt.} \propto N_i S_i.$$

In this connection we also refer to an early work of Kovalevsky (1924).

The above allocation does not reflect costs involved and it was Mahalanobis, who even in thirties introduced the cost function of the type (see, Mahalanobis,1944)

$$C = C_0 + \sum_{i=1}^k c_i n_i \quad (3.1)$$

where C is the total cost, C_0 is the fixed overhead cost and c_i is the sampling cost per unit and n_i is the sample size in the i th stratum, $i = 1, 2, \dots, k$. He then obtained an allocation that minimizes $\text{Var}(\hat{Y}_{st.})$, the variance of the estimate of the population mean under SRS in all strata subject to the cost restriction given in (3.1) resulting in the allocation

$$n_i^M \propto N_i S_i / \sqrt{c_i} .$$

A more mathematical theory for inference within a finite population frame work was developed in Neyman(1937), while practical surveys of large human populations were considered in Neyman(1938) introducing cost functions for double sampling technique among other things.

At Rothamsted Experimental Station, Yates and Zaccapony(1935) and Cochran (1939,1940) developed sampling methods following Fisher's principles of experimental designs. Analysis of Variance was adopted while estimating the yields of cereal crops. Multistage sampling with first stage units of equal size was also thought of (cf. Hansen(1987) for details).

Simultaneously in India, during the forties, while Mahalanobis was organising surveys at the Indian Statistical Institute, Sukhatme was responsible for conducting crop cutting surveys, surveys in Biometry and Animal Husbandry at the then Imperial Council of Agricultural Research(ICAR). Around this time, the attempts at various institutions materialised into a sound mathematical theory of several techniques in probability sampling and important contributions appeared in the form of text books and publications on topics such as probability proportional to size (pps) sampling, systematic sampling, stratified sampling, ratio and regression methods of estimation, multi stage and multi phase sampling, rotation schemes, non sampling errors etc.,(see later sections). Reviews prior to 1952 appeared in Mahalanobis(1946), Yates(1946), Stephan(1948) and Seng(1951).

4. A brief look at the later developments

The turning point that prompted a rethinking in inference based on finite population sampling came from Horvitz Thompson's 1952 work and Godambe's 1955 non existence theorem. (referred to in latter sections).

Design based inference was the theme of many results until Royall(1970) and his co authors introduced the concept of model unbiasedness in place of design unbiasedness which made ratio estimator more relevant and Horvitz Thompson estimator took the back stage. Balanced samples and model based inference (model assisted survey sampling) were topics of research. Modelling was used by earlier authors, but a useful approach was first given by Brewer(1963) which became more popular and accepted with Royall's work and the Generalized Regression (GREG) estimator of Cassel, Särndal and Wretman(1976). During the recent years, it was Brewer(1999) again who has bridged the two approaches of design-based and prediction-based inferences.

Of reviews that appeared recently, we refer to Kuusela(2011) for certain paradigms in finite population inference and Brewer(2013) for suggesting a conciliation of opposing approaches mentioned above.

5. Basic Sampling Techniques

5.1 Simple Random Sampling

S. Raja Rao of, the then, National Sample Survey(NSS) wing of the Indian Statistical Institute(ISI) seems to have observed in the fifties that for SRS with replacement, the mean based on the distinct units which is also unbiased for the population mean, exhibited smaller variance compared to the mean based on all the units of the sample. Hanurav(1965) in his thesis and Chaudhuri(2014) refer to this observation and note that Raja Rao consulted with D. Basu of the Research and Training School of ISI. Basu with his profound expertise in statistical inference immediately noted that the 'order statistic' based on the distinct units in SRSWR is a sufficient statistic and invoked the sufficiency principle in sampling theory. Raja Rao thus collaborated with Basu in establishing the superiority of the mean of distinct units in SRSWR over the conventional estimator based on all units drawn. Basu(1958) extended the result to other sampling designs as well. Thus the concept of 'sufficiency' in sampling theory was the brainchild of Basu and later studied by several researchers.

5.2 Probability Proportional to Size(PPS) Sampling.

Even in 1937, much before Hansen and Hurwitz(1943) published their paper on PPS sampling, Mahalanobis was aware of this technique as seen from the report on experimental crop census conducted in 1937. For

obtaining valid estimates, he suggests the selection of plots using the cumulative totals of their areas, since the areas had a wide variation. However, due to the extra manual workload and the travel costs involved, he found the cumulative total method of PPS impracticable and recommended grids.

5.3 Systematic Sampling

Mahalanobis, Majumdar and Rao who organised the United Provinces (U.P.) Anthropometric Survey in 1941 (MMR,1949) describe their method of selection of material as follows:

“The random selection of sample is not an easy task. To pick up a random sample in a demonstrably rigorous fashion requires elaborate preliminary arrangements which are not often possible in practice. In this situation, rough and ready methods have to be used”.

Majumdar, the anthropologist who was taking the measurements

“collected all healthy males between the ages of 18 and 48 (belonging to the caste or tribe under survey) who happened to be available, arranged them in serial order just as they came and picked up either the odd or even numbered individuals for measurement”.

Summing up, MMR claim:

“the present samples may therefore be treated as having been drawn, for all practical purposes, at random. As far as one can judge, the assumption of randomness is more true of the present material than any of other series of anthropometric measurements so far available in India.”

Majumdar's selection method can be thought of as using a random start from 1 and 2 and selecting a systematic sample with a sampling interval of two, a technique which was developed a few years later by Madow and Madow(1944). The arrangement of individuals 'in a serial order as they came in' makes systematic sampling equivalent to simple random sampling as claimed by MMR above. It is now known that Madow and Madow(1944) justify this concept showing that systematic sampling is, on the average, equivalent to simple random sampling if the order of the units in a specific finite population can be regarded as drawn at random from all permutations of the units of the population. This observation of Madow and Madow led to the concept of 'random

permutation (RP) model' for drawing inferences from finite populations. Thus a few years before Madow and Madow's justification, MMR unknowingly guessed that what they were doing is sound *for all practical purposes*. It is only about three decades later that C.R. Rao(1971) turned his attention to, by then the well known concept of inference using RP models.

It may be recalled that when Bowley conducted the Reading survey on the general economic condition of working class, he took 'one building in ten' from the local directory in an alphabetical order of streets, which amounts to something like a systematic sample.

6. Analysis of data from Complex Surveys and Small Domain Estimation--antecedents

Following the U.P. Survey of 1941, Bengal Anthropometric Survey was organised in 1945. In the statistical assessment of A-B-O blood group data on Bengalees discussed in Majumdar and Rao(1958), C.R. Rao noticed that *the investigator has to travel from place to place and collected a few individuals belonging to a particular caste in each place, which amounted to stratified sampling and not simple random sampling*. Thus C.R. Rao was perhaps the first to recognise that such data needed an alternative analysis (Skinner *et al.*, 1989) which came to be known as analysis of complex survey data. J.N.K. Rao(2005) in his appraisal mentions the rapid progress that was made in this direction .

In the Foreword of the paper referred to above on Bengal Survey, Mahalanobis defines 'group' as individuals belonging to the same caste, religion or tribe in the same district, giving rise to a two way classification. He then goes on to say that 'any further sub-division on the basis of sub-caste, clan, endogamy etc. would have resulted in a large number of such groups for which a much larger survey would be necessary to obtain sufficient number of individuals under each sub-group for proper statistical analysis'. The same problem arises in theory of estimation for small domains(areas) . For the past several decades, this subject attracted the attention of theoreticians as well as practitioners of sample surveys(see, J.N.K. Rao(2003)) in view of the need for estimation at decentralized levels.

7. Utilisation of auxiliary information in sample surveys

7.1 Utilisation for allocation of sample size in stratified sampling.

We have introduced earlier the problem of allocation of sample size in stratified sampling due to Tschuprow – Neyman. However, here S_i^2 are

unknown parameters and during the discussion on Neyman's 1934 JRSS paper, the question of how the suggested optimum allocation could be applied in practice was not raised. It was Sukhatme who in the following year gave the following practical solution for using this allocation. (Sukhatme, 1935):

Draw an initial sample of size m , estimate S_i^2 as \hat{S}_i^2 and use

$$n_i^{near\ opt.} = n N_i \hat{S}_i / \sum_{i=1}^k N_i \hat{S}_i .$$

He also compared its efficiency with Bowley's proportional allocation.

7.2 Utilisation in PPS Sampling without replacement

7.2.1 Horvitz Thompson estimator

We have already seen in Section 5 on PPS sampling, how a related auxiliary variable, namely area of plot is used to estimate the crop yield for greater efficiency. We shall now turn our attention to PPS sampling without replacement.

Horvitz and Thompson were the first to formulate three classes of linear estimators for the finite population total in accordance with a unified approach depending upon the draw, unit or the sample. In their paper (Horvitz and Thompson(1952)), they considered their T_2 class of estimators defined by $\hat{T}_2 = \sum \beta_i y_i$, where β_i is a constant to be used as weight for the i th. unit whenever it is selected in the sample. They showed that the only unbiased linear estimator possible in this sub class is the one with $\beta_i = 1 / \pi_i$, where π_i is the probability of inclusion of the i th unit in the sampling design. Thus we have the celebrated Horvitz Thompson (HT) estimator which is the only unbiased estimator in this class given by

$$\hat{Y}_{HT} = \sum_{i \in s} y_i / \pi_i .$$

Horvitz and Thompson (HT), indeed recognize that this is the "*only unbiased linear estimator possible in the subclass under consideration and hence is 'best' for that subclass*". They also derive an expression of the $\text{Var.}(\hat{Y}_{HT})$ and suggest an unbiased estimator of this Variance. One notes from their paper the following:

"... *It is the opinion of the authors that the techniques suggested by this paper may be of greatest utility in specialized enquiries where the*

characteristics under measurement are few and related, or where selection with unequal probability arises naturally.”.

HT observed that when π_i is proportional to Y_i , their estimator has zero variance and the sampling will be optimum. When auxiliary information on a related characteristic \mathcal{X} is available taking values X_i on units U_i , $i= 1, 2, \dots, N$, then $\pi_i = n P_i$, with $P_i = n X_i/ X$, where $X = \sum X_i$ would be near optimum.

Thus in PPSWOR sampling for $n = 2$, HT solved for ‘working probabilities’ Q_i associated with U_i , so that π_i based on these is equal to $2 P_i$. An year later, Yates and Grundy(YG, 1953) also suggested alternative working probabilities and obtained the solution for $n = 2$ in the case of PPSWOR. In Yates and Grundy (1953), Yates notes that :

"The present investigation was originally undertaken by the first author (Yates) with the object of determining in more detail the errors likely to result from this procedure of selection and estimation" "It was only after the paper had been prepared and submitted for publication that a copy of a recent paper by Horvitz and Thompson (1952) on the same subject became available ... ”

From this, one may thus conclude that HT estimator for $n= 2$ was also independently obtained by Yates around the same time and credit also goes to Yates. A forerunner to these results is Narain’s (1951) solution of working probabilities where he brought “ *certain features novel to sampling without replacement*”.

Acknowledging this , Yates and Grundy(1953) mention:

“..Narain (1951) has given an alternative solution of which we are unaware until after this paper had been sent to press...”.

Perhaps, this estimator should be termed as ‘Narain-Yates-Horvitz-Thompson’ estimator. J.N.K. Rao(1999) has already called it the Narain-Horvitz-Thompson estimator.

Furthermore, we note from Yates and Grundy’s (1953) paper the following:

"... Although Horvitz and Thompson's paper deals with much the same problem, the conclusions reached by the first author (Yates) differed from theirs in many respects..." . "In one respect, Horvitz and Thompson took the matter further in that they gave an unbiased estimator of the error variances. This however, proved on examination to be unsatisfactory... (negative for some samples). This fact stimulated the second author

(Grundy) of the present (revised) paper to search for a better unbiased estimator which is included in the revised version ...".

Rewriting the variance expression of HT for a fixed sample size(n) sampling design as a double summation, Grundy in their paper gave an unbiased estimator of the above variance . Referring to its non negativity, the authors claim "*this appears to be the case when the usual method of selection is employed*". However, their variance estimator also turns out to be negative but 'less often' than HT's variance estimator .

Sen(1953), not only quite independently derived this variance estimator , but also supplemented his results by obtaining sampling schemes for which the 'Grundy variance estimator' is non negative. Thus what is known in the literature as 'Yates-Grundy variance estimator' or 'Sen-Yates-Grundy variance estimator' could be called as 'Sen-Grundy variance estimator' in view of the above mentioned quote .

7.2.2 Das's estimator.

In his 1951 paper, Das(1951) mainly considered two-phase sampling scheme and states in the summary :

".....besides this, the method of analysis in the case of sampling with varying probabilities is also discussed".

It seems that the latter scheme was only of a passing interest to Das. He noted that in sampling without replacement "*it is necessary that the order*" of the individuals coming in the sample "*should be known*". With the usual notation, Das's unbiased estimators for the population total Y are given by:

$$t_1 = y_1 / p_1$$

$$t_2 = (1 - p_1) y_2 / (N-1) p_2 p_1,$$

$$t_3 = (1 - p_1 - p_2) (1 - p_1) y_3 / (N-1)(N-2) p_3 p_2 p_1$$

.....

and so on.....

He then suggests using the simple unbiased estimator $\sum t_i / n$ and also obtains an estimator of the variance. Das's estimator has been neglected (Rao,2015) in the sampling literature. The main defect seems to

be the negative value taken by the variance estimator for some samples. One notes that even HT estimator of variance of HT estimator for total Y or Sen-Grundy variance estimator turns out to be negative. The beauty of Das's estimator is that at any stage of draw of the sample, it is enough to know the y value of that selected unit. There is no need to know the previous y values. It may be a case of 'missing data' or 'unrecorded data'(due to non response/negligence of the investigator etc..) Here p_i 's based on auxiliary information are already known. One feels that this is a positive aspect compared to HT estimator or Des Raj estimator or symmetrized Des Raj estimator (next section). We may call this as the 'face validity' , a 'difficult to define' concept *a` la* Basu (1971).

7.2.3 Des Raj's estimator

Des Raj presented three sets of unbiased estimators for Y depending on the order of selection of units (without replacement) . Of these, the first set is the oft quoted (Des Raj,1956) one given by:

$$t_1 = y_1 / p_1$$

$$t_2 = y_1 + (y_2 / p_2) (1 - p_1)$$

$$t_3 = y_1 + y_2 + (y_3 / p_3) (1 - p_1 - p_2)$$

..... and so on.

Here unlike Das's estimator, to calculate the estimator at r th. draw, one needs to know all the y values of the previous draws. However, the greatest advantage of this set of estimators is that they are simple and since t_i 's are uncorrelated , a simple non negative variance estimator of the estimate \bar{t} of the total is available .

The second set of estimators is the same as the one already given by Das(1951) discussed above, with no reference to Das(1951). This might have been reported by Des Raj for the sake of completeness. The only additional remark is that the variance estimator (same as the one given by Das, but simpler) may assume negative values . Das's paper being a conference paper could have been overlooked. Discussing Yates and

Grundy's estimator of variance, it is mentioned that the results " *have also been noticed by Sen*". There is an interesting second part of the Des

Raj's paper wherein, the Des Raj's and Das's estimators are extended to multistage designs.

7.2.4 Murthy's estimator

In Murthy(1957), it is shown that “ *corresponding to any biased or unbiased ordered estimator, there exists an unordered estimator which is more efficient than the former .*” The technique of improving the ordered estimators by unordered ones is applied to Das's and Des Raj's sets of estimators. For sampling of two units by PPSWOR, Murthy writes expression (3.17) of his paper as the unordered (Symmetrized Des Raj's estimator) given by (with the usual notation):

$$\bar{t}^{SDR} = \{ (1 - p_2) (y_1 / p_1) + (1 - p_1) (y_2 / p_2) \} / (2 - p_1 - p_2) .$$

In a footnote (p.384), Murthy says “ *Lahiri conjectured that Des Raj's estimators can be improved by weighting the different ordered estimators by their respective probabilities and in fact suggested the estimator given in (3.17).* In view of this, one tends to call this much cited estimator as ‘Lahiri-- Murthy estimator’ rather than ‘Murthy's estimator’ . This seems to have escaped the attention of researchers for over six decades in spite of Murthy's footnote. In his paper, Murthy (1957) also had a discussion on the variance estimators . Only for one particular scheme , where the first unit is selected with PPS and the remaining $(n-1)$ units are selected by SRSWOR , it is shown that Das's unordered estimator becomes the unbiased ratio estimator (see section 7.7). Pathak(1961) improved the ordered estimators at r th draw of both Das's and Des Raj's to corresponding unordered estimators by conditioning on the order statistic (sample units arranged in ascending order of their unit indices, which is a sufficient statistic) . Though he gets the same Lahiri-- Murthy estimator for $n = 2$, he gave a general expression without much simplification. As we said before, not so much work was done on unordering of Das's estimator as compared to unordering of Des Raj's.

7.3 Utilisation in ratio method of estimation

Ratio method of estimation probably dates back to the attempts made by Graunt(1662) for estimating the population of London, who according to Brewer (2013) used a somewhat vague notion of a ratio estimator.

It is said that assuming an initial number of 12,000 annual births, Graunt multiplied this by two to get 24000 fertile women population and

again using a factor of two concluded that there are 48,000 families . Then with a assumed family size of eight, calculated the population of London as 384,000. The other justification he had was based on an observed death rate of 3 out of 11 in a parish from London registries of that parish, and knowing that there were 13,000 burials , estimated the number of families as $143000/3 \sim 48,000$ and again with the crude estimate of family size eight, calculated the population of London as 384,000(see, Sen(1993) for details). Brewer (2013) notes that the notion of a ratio estimator was there in Graunt's work which probably what a serious survey statistician would not call it so.

Brewer (2013) further realises that what Graunt employed is nothing but the 'rule of three'. It is interesting to point out that Rao (2010b) comments on the influence of C.R. Rao on theory and practice of sample surveys thus:

"...He(C.R.Rao) would make concepts very simple and easily understood , for example : Ratio method for estimating population total is introduced as 'rule of three'...."

Messance (1766) and Moheau (1778) were credited with the use of ratio method for the estimation of the population of France. Their ratio estimator is obtained by multiplying the ratio of number of persons based on enumeration of population in certain sample districts to the number of births and deaths in those districts by the number of births and deaths for the entire country.

Later on, Laplace(1786) made certain assumptions and modifications of the above methods besides calculating the margin of error for his estimate and the sample size needed to attain the required degree of accuracy in estimation. Laplace's work on the large sample distribution of the ratio estimator and his concept of 'super population' are considered by Cochran(1978) as 'pioneering achievements'.

Though these methods had their own deficiencies, we note that these are the first attempts of using auxiliary information on a related characteristic to obtain a stable estimate of the population total (for details , we refer to Sen(1993), Brewer(2013) and Kuusela(2011)). It may be pointed out that Laplace while estimating the population of France assumed that France itself and the sample of communes were indeed binomial samples from an infinite superpopulation (cf. Cochran,1978).

7.4 A revisit to 'ratio estimation'

Recent activity centred much on this old technique of ratio estimation once again. What is well known as 'the ratio estimation property'(for

example, see J.N.K. Rao(1979)) was given a boost by the work of Deville and Särndal (1992) who termed it ‘calibration’ which is found to be very useful in practice. Owen(2001) considers this as similar to ‘empirical likelihood’. Almost five decades ago, Hartley and Rao(1968) provided an alternative ‘scale load’ approach suggesting a well defined ‘likelihood’ which makes the conventional theory amenable to apply, in view of the otherwise ‘flat’ likelihood considered by Godambe and others. Owen comments thus:

“.....Hartley and Rao(1968) also provide what may be the very first MELE, maximizing a continuous version of the likelihood subject to a constraint on the mean...”

While Owen(1991) is credited with the use of calibration for *iid* observations, Deville and Särndal (1992) used calibration weights depending on inclusion probabilities of units which opened up several avenues such as ‘pseudo empirical likelihood’(cf. Chen and Sitter(1999), Wu and J.N.K. Rao(2006)) and further extensions ((for example, see Kim(2009)). Rao and Sengupta(2010) demonstrated that from an initial biased homogeneous calibrated linear estimator, it is possible to obtain unbiased hle’s which still are calibrated with respect to the same auxiliary variable. An interesting information-theoretic perspective was given in Wittenberg(1999) . Econometric application to Madagascar economy was considered by Robilliard and Robinson (2003).

7.5.Utilisation in product method of estimation

The dual of ratio method of estimation was proposed by Murthy(1964) when the study variable and auxiliary variable are negatively correlated. If \hat{Y} and \hat{X} are unbiased estimators of population totals Y and X respectively , then a product type estimator for the population total is given by

$$\hat{Y}_p = \hat{Y} \hat{X} / X .$$

Robson (1957) was the first to introduce the product estimator , but not referenced in Murthy(1964). While Robson used multivariate polykays, Murthy used independent subsamples approach and thus the contents of their papers are different.

7.6 Multiple regression estimator

Historically, data on several auxiliary variables for inference was used in purposive selection method. A successful study in Denmark where ten

auxiliary variables are utilized is mentioned in Jessen(1926) while a complete failure of purposive selection in Gini and Galvani's(1929) balanced samples based on several variables is well known. For measuring the yield of Cinchona bark, Mahalanobis in 1940 used three physical measurements for regressing(Mahalanobis,1946). Towards the end of fifties, Olkin(1958) extended the ratio estimator to the case where data on p auxiliary variables is available. However, we note that it was Ghosh(1947), a decade earlier, who first envisaged the concept of double sampling with many auxiliary variables.

It was pointed out earlier(Rao,1993) that some authors erroneously combine a ratio estimator and a product estimator both based on the *same* (single) auxiliary variable or combine ratio and product estimators based on several auxiliary variables without regard to their correlation with the study variable.

7.7 Unbiased ratio estimation.

Consider the ratio type estimator for the population total Y of a study variate y when auxiliary information on a related characteristic x is available whose population total is X , given by

$$\hat{Y}_{\text{ratio}} = (\sum_S y_i / \sum_S x_i) X. \quad (7.7.1)$$

This is biased for Y for SRS design and Lahiri (1951) suggested the sampling design

$$p(s) = \sum_S x_i / (N - 1 \binom{N-1}{n-1}) X$$

which makes the above ratio estimator unbiased for Y . Lahiri noted that the sum over all (N_{C_n}) samples of the aggregate sample total in the numerator gives the denominator and observed that the design is nothing but a probability proportional to aggregate size sampling design. Since the cumulative (aggregate) total method will be difficult to adopt, he devised the technique, which is the original 'Lahiri's Method'. When the sample size is unity, we have N units with the corresponding sizes X_i , $i=1,2,\dots,N$ and this is referred to as Lahiri's method of PPS selection in some text books. (Here (N_{C_n}) reads : N choose n).

Around the same time, as attributed by Midzuno(1952), Ikeda(1950) worked out a selection procedure for a particular case of this problem while Midzuno (1952) and Hanjeka(1949) gave the following simple scheme of achieving this design as follows:

Draw the first unit with probability proportional to size and the remaining $(n-1)$ units by SRS Without Replacement from the rest of the $(N-1)$ units of the population.

Sen considered the same problem as early as February 1950 and has included the solution in his thesis submitted to the University of North Carolina in January 1952. We quote from Sen(1993) :

“...Neither was he aware of the results of Lahiri(1951) working in India nor of Midzuno(1952) in Japan and Ha'jek(1949) in Czechoslovakia...”

Sen (1952,1953) also went on to prove that , for $N \geq 3$ and $n = 2$, when the first unit is selected with probability proportional to size (PPS) and the second unit by PPS from the remaining $N-1$ units, Sen-Grundy variance estimator is non negative. He also demonstrated that Sen-Grundy variance estimator is non negative for the above mentioned scheme of selecting a sample of size n .

This result thus may be termed as ‘Lahiri-Ikeda-Midzuno-Sen-Ha'jek’ sampling scheme , popularly known as Midzuno-Sen scheme or Lahiri-Midzuno-Sen scheme in most of the text books.

The above paragraphs describe a method of constructing a sampling design(scheme) that makes a given ratio estimator { here the ratio of means (namely $t_1 = (\bar{y} / \bar{x}) X$, an equivalent form of (7.7.1))} unbiased for Y . Alternative methods of constructing unbiased ratio estimators involve combination of the two biased estimators for Y , namely ratio of means t_1 and mean of ratios $t_n = (X/n) \sum_1^n (y_i/x_i)$.

It is observed that unbiased ratio estimators obtained this way such as Murthy- Nanjamma's(1959), Hartley- Ross's(1954) or Neito de Pascual's (1961) are either non convex combinations or non stochastic convex combinations of t_1 and t_n . While a non convex combination of t_1 and t_n is given by $\lambda t_1 + (1-\lambda) t_n$, where λ does not belong to $(0,1)$, a non stochastic convex combination is given by $\lambda t_1 + (1-\lambda) t_n$, where λ is a random variable and $E(\lambda)$ does not belong to $(0,1)$. This concept was found useful in identifying unbiased ratio estimators (see Rao,2010a).

8. Unified Theory of Sampling

8.1 Super population models.

Godambe(1955) defined the most general type of homogeneous linear estimator(h.l.e.) for the finite population total $Y = \sum Y_i$ of a study variate \mathcal{Y} taking values Y_i on the i th. unit, of which Horvitz- Thompson's (1952) three classes are particular cases. He then established the well known nonexistence theorem which proves the nonexistence of a unique homogeneous linear unbiased and least variance estimate for all values of \underline{Y} . In view of this non existence, Godambe suggests that the “*next best thing that we can do is to search for a procedure of estimation which when employed repeatedly would secure on the average a least variance*”. Thus he employs a ‘super population’ concept relating to the *apriori* expectations of \underline{Y} . (also see, Cochran(1939,1946), Deming and Stephan(1949), Yates(1949) and Deming(1953) where the finite population is considered as a sample from an infinite one . As noted earlier, Laplace , while estimating the population of France is believed to have used the concept of super population).

The sampling strategy consisting of a fixed sample size(n) design with inclusion probabilities proportional to size measure (π PS or IPPS design) together with Horvitz Thompson estimator is proved to be optimum in the sense of having minimum expected variance (Godambe,1955) under an assumed super population model. This confirms HT's claim of optimality of their estimator. This result opened up the floodgates and several sampling schemes were published by researchers . An excellent monograph first released in this context was by Brewer and Hanif (1983).

8.2 Unicluster designs.

A decade after Godambe(1955) proved his non existence theorem that there does not exist a uniformly minimum variance unbiased estimator in the general class of sampling designs, Hanurav (1965) in his thesis showed the existence of exceptions. These are called ‘unicluster designs’ for which two samples are either identical or are disjoint(systematic sampling design being one such) . The same exceptions were noted by Hege (1965) also independently.

9. Non sampling errors.

9.1 Mahalanobis and early survey practice

The three notable contributions of Mahalanobis to the theory and practice of sample surveys for reducing the sampling errors, and assessing and controlling the non sampling errors are ‘pilot surveys’, ‘optimum survey design’ and ‘Inter Penetrating Network of Subsamples’(IPNS) technique. Referring to the pilot surveys of Mahalanobis, Wald(1947) in the Preface of his book on ‘*Sequential Analysis*’ acknowledges thus:

“ The occasional practice of designing a large scale experiment in successive stages may be regarded as a forerunner of sequential analysis. A very interesting example of this type is a series of sample censuses of area of jute in Bengal carried out under the direction of Mahalanobis ”.

Regarding the second contribution, Hansen, Hurwitz and Pritzker (1964) remarked that “ Mahalanobis had a broad view of the task the statistician engaged in statistical surveys....In stressing that all the resources provided for a survey should be used optimally,....went beyond the mathematically tractable proposition that the sampling error should be minimized for fixed cost (or that the cost should be minimized for a fixed sampling error.)...”

Acknowledging Mahalanobis’s IPNS technique, Deming(1964) mentions: “....For 14 years I have used only interpenetrating network of samples, initiated by him (Mahalanobis), as every one knows, about 1936.....”. This has become the curtain raiser for the much used ‘resampling procedures’.

9.2 Supervision.

To control non sampling errors, among other things, supervision during the field work is thought to be very important. Mahalanobis, the chief architect of post independence statistical system in India, who also initiated the National Sample Survey(NSS) in 1950, introduced an independent supervisory staff during the field operations of NSS for collection of reliable data.

In Kautilya’s *Arthashastra* (attributed to 321-296 B.C.), among other data collection approaches, the need for cross checking by an independent set of agents working under disguise was mentioned . It is said that Mahalanobis was probably influenced by this ‘striking feature’ in the *Arthashastra*.

Coincidentally, Hansen at the United States Bureau of Census also had a supervisor, called 'crew leader' at an intermediate level. In a conversation with Olkin published in *Statistical Science*, 1987,p.172, Hansen himself sums up : “.....Each (Hansen at US Bureau of Census and Mahalanobis at Indian Statistical institute) was looking at total sample survey design and using innovative approaches and developing theory as needed. None of us was developing theory to write papers. In fact, we didn't write many papers. But when you have a problem to solve and you need some theory, you develop the theory to solve the problem .”

9.3 'Not-at-Homes'.

Non response is a major problem in sample surveys contributing to non sampling errors. Two of the sources of non response are 'not at homes' and refusal to answer sensitive questions. For the first category of 'not at homes', the technique of 'call-backs' was proposed. Initial attempts to control this type of non response are due to Yates(1946) and Hansen and Hurwitz(1946). During the fifties several developments took place in relation to call-backs and the relative costs that are involved(see Deming(1953), Durbin and Stuart(1954), Stephan and McCarthy(1958), Kish and Hess(1959)) among others.

During the discussion of the paper read by Yates(1946) at the RSS Meeting, Hartley(1946) proposed an 'ingenious' and 'decidedly cheaper' alternative to call-backs . Hartley mentions:

“Details of this scheme were given to the War-time Social Survey, but I understand that, owing to pressure of work, an opportunity of trying has, as yet, not arisen”.

Soon after, Politz and Simmons (PS,1949) published their work popularly known as Politz-Simmons technique in the *Journal of American Statistical Association* which is on similar lines to the proposed method of Hartley. PS (1949) while acknowledging the work of Hartley , say:

“It has recently been brought to the authors' attention that a somewhat similar plan was proposed independently by H.O.Hartley before the Royal Statistical Society.....

.....During the past three years we have developed a plan for eliminating the need for call-backs and several experiments have been made applying this plan to market surveys.”

This technique consists of grouping the responses based on first (only one) call as follows:

Assume that all interviews are carried out during the evening on the six weeknights. The respondent is asked whether he was at home at the same time on each of the five preceding weeknights. If the number of times he was at home is i out of five, then $(i + 1)/6$ is an estimate of the probability of the respondent being at home on a randomly chosen evening. The values of the characteristic grouped will get the weights based on these estimated probabilities.

In view of the above discussion, we term this as ‘Hartley-Politz-Simmons’ (HPS) technique(cf. Rao,2014; Rao, Sarkar and Sinha(RSS, 2016)) rather than the oft quoted ‘Politz –Simmons’ technique.

Noting that “*for reasons of modesty, fear of being thought bigoted, or merely a reluctance to confide secrets to strangers, many individuals attempt to evade certain questions put to them by interviewers*”, Warner(1965) introduced the innovative concept of ‘Randomized Response Technique(RRT) to elicit information on sensitive (stigmatizing) questions. During the last five decades vast amounts of literature has been published on this topic , a recent addition being *Handbook of Statistics*, Volume 34 , released by Elsevier in 2016.

In the context of HPS technique mentioned above, the question posed before the respondent is about his/her presence at home at the same time of interview during the preceding ‘five weeknights’. In the present day context of the ever changing society, we note that this question is sensitive and respondents evade an answer or give a wrong answer so as to guard their privacy for security reasons, personal movements, family interactions and such similar situations. To tackle this problem, RSS(2016) discussed a ‘Randomized HPS technique’.

Conclusion

In the above sections, we have outlined several early innovative techniques in the theory and practice of sample surveys. On certain occasions, researches have not paid attention to the chronology or priority. This could have happened because some nations were involved in wars or calamities or political instability thereby disturbing the academic atmosphere. In some other situations, it was the language barrier and absence of proper interactions. The aim of this paper, as mentioned before, is to place on record the innovative advances that were made independently (pre -www era) by different authors and give credit where it is due. It is expected with search engines and web available now, there will be few duplications or cases of priorities. As mentioned before, this is not a review of recent topics in sampling theory

and practice but concentrates on the history of certain key concepts which was overlooked.

We have not covered priorities relating to establishing the superiority of one estimator over another, or simple extensions or generalizations and such. We have concentrated only on early key concepts. It may be noted that it is difficult to cover all such aspects and in this paper we have tried to outline the early innovative topics which involve most of the key concepts.

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